1. Three-Point correlation (Coins):

In deriving the equation for single point distribution f_1 and $g(1,2,t)$ we defined the joint probability f_3 in terms of the one-point probability f_1, the two-point correlation function g, and the three-point correlation function h. We apply this to the case of three coins, each of which can come up with heads (+) or tails (-). What is the meaning of f_3 in this case? Write out f_2 and f_3 in the form

$$
\begin{aligned}
f_2(1,2) &= f_1(1) f_1(2) + g(1,2) \\
f_3(1,2,3) &= f_1(1) f_1(2) f_1(3) + f_1(1) g(2,3) + f_1(2) g(1,3) + f_1(3) g(1,2) + h(1,2,3)
\end{aligned}
$$

and evaluate f_3, f_1, g, and h in each of the following cases.
(a) All three coins are “honest”, that is, each coin is equally likely to come up heads or tails, and each coin is unaffected by any other coin.
(b) Because the coins are mysteriously locked together, in any one throw all three are heads.
(c) The first coins two are honest but 3 is manipulated to show heads if 1 or 2 are heads. How is your chance changing if you bet on ttt and can you see this looking at the two- or thee-point correlation?

Hint: Characterize the states of the random single point distribution with the Kronecker δ as $f_1(1) = 0.5(\delta_{1h} + \delta_{1t})$. Make use of the relations $\delta_{1h} + \delta_{1t} = 1$ (because there are only two states) to manipulate/simplify your results.

Solution:

(a) Honest coin: $f_1(1) = 0.5(\delta_{1h} + \delta_{1t}) = 1/2$

Two coins: $f_2(1,2) = 1/4 = f_1(1) f_1(2) + g(1,2)$ => $g(1,2) = 0$

Three coins:

$$
\begin{aligned}
f_3(1,2,3) &= 1/8 \\
&= f_1(1) f_1(2) f_1(3) + f_1(1) g(2,3) + f_1(2) g(1,3) + f_1(3) g(1,2) + h(1,2,3) \\
&= 1/8 + h(1,2,3)
\end{aligned}
$$

=> $h(1,2,3) = 0$

(b) All three coins are heads: $f_1(i) = \delta_{ih}$ for $i = 1,2,3$.

Two-point propability: $f_2(1,2) = \delta_{1h}\delta_{2h}$ because the result is always hh. From $f_2(1,2) = f_1(1) f_1(2) + g(1,2)$ it is immediately obvious that $g(1,2) = 0$ and the same for any other choice of indices $g(1,3) = g(2,3) = 0$.

Two-point propability: $f_3(1,2,3) = \delta_{1h}\delta_{2h}\delta_{3h}$ because the result is always hhh. From $f_3(1,2,3) = f_1(1) f_1(2) f_1(3) + f_1(1) g(2,3) + f_1(2) g(1,3) + f_1(3) g(1,2) + h(1,2,3)$, and $g(1,2) = 0$ it is immediately obvious that $h(1,2,3) = 0$.

(c) First two coins are honest but 3 is heads if 1 **or** 2 is heads. First coin is honest -> $f_1(1) = 1/2 = f_1(2) = f_1(3)$

Two coins (distribution is not symmetric):

$$f_2(1,2) = 1/4$$

because 1 and 2 are honest and there are 4 outcomes each with the same probability of 1/4. For

$$
\begin{aligned}
f_2(1,3) &= \frac{1}{2}\delta_{1h}\delta_{3h} + \frac{1}{4}\delta_{1t} = \frac{1}{2}\delta_{1h}\delta_{3h} + \frac{1}{4}(1 - \delta_{1h}) \\
&= \frac{1}{4} + \frac{1}{4}\delta_{1h}\delta_{3h} - \frac{1}{4}\delta_{1h}\delta_{3t}
\end{aligned}
$$

because there is a probability of 1/2 that 1 comes up h (which fixes $3 = h$) and if 1 is tails there are two possibilities for 3, i.e., 3=h and 3=t each with a combined probablity of 1/4. In summary

$$
\begin{aligned}
f_2(1,2) &= 1/4 = f_1(1) f_1(2) + g(1,2) = 1/4 + g(1,2) \\
f_2(1,3) &= 0.5\delta_{1h}\delta_{3h} + 0.25\delta_{1t} = 1/4 + g(1,3) \\
f_2(2,3) &= 0.5\delta_{2h}\delta_{3h} + 0.25\delta_{2t} = 1/4 + g(2,3) \\
such\ that & \\
g(1,2) &= 0 \\
g(1,3) &= -\frac{1}{4} + \frac{1}{2}\left(\delta_{1h}\delta_{3h} + \frac{1}{2}\delta_{1t}\right) = -\frac{1}{4} + \frac{1}{4}\left(2\delta_{1h}\delta_{3h} + 1 - \delta_{1h}\right) \\
&= \frac{1}{4}\left(2\delta_{1h}\delta_{3h} - \delta_{1h}\right) = \frac{1}{4}\delta_{1h}\delta_{3h} + \frac{1}{4}\delta_{1h}\left(\delta_{3h} - 1\right) = \frac{1}{4}\delta_{1h}\delta_{3h} - \frac{1}{4}\delta_{1h}\delta_{3t} \\
g(2,3) &= \frac{1}{4}\delta_{2h}\delta_{3h} - \frac{1}{4}\delta_{2h}\delta_{3t}
\end{aligned}
$$

3 coins:

$$
\begin{aligned}
f_3(1,2,3) &= \frac{1}{4}\delta_{1h}\delta_{2h}\delta_{3h} + \frac{1}{4}\delta_{1t}\delta_{2h}\delta_{3h} + \frac{1}{4}\delta_{1h}\delta_{2t}\delta_{3h} + \frac{1}{8}\delta_{1t}\delta_{2t}\left(\delta_{3h} + \delta_{3t}\right) \\
&= \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3h} + \frac{1}{8}\delta_{1t}\delta_{2h}\delta_{3h} + \frac{1}{8}\delta_{1h}\delta_{2t}\delta_{3h} + \frac{1}{8}\delta_{1t}\delta_{2t}\left(\delta_{3h} + \delta_{3t}\right) \\
&\quad + \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3t} + \frac{1}{8}\delta_{1t}\delta_{2h}\delta_{3t} + \frac{1}{8}\delta_{1h}\delta_{2t}\delta_{3t} \\
&\quad + \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3h} + \frac{1}{8}\delta_{1t}\delta_{2h}\delta_{3h} + \frac{1}{8}\delta_{1h}\delta_{2t}\delta_{3h} - \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3t} - \frac{1}{8}\delta_{1t}\delta_{2h}\delta_{3t} - \frac{1}{8}\delta_{1h}\delta_{2t}\delta_{3t} \\
&= \frac{1}{8} + \frac{1}{8}\delta_{2h}\delta_{3h} + \frac{1}{8}\delta_{1h}\delta_{3h} - \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3h} - \frac{1}{8}\delta_{2h}\delta_{3t} - \frac{1}{8}\delta_{1h}\delta_{3t} + \frac{1}{8}\delta_{1h}\delta_{2h}\delta_{3t} \\
&= \frac{1}{8} + \frac{1}{8}\left(\delta_{3h} - \delta_{3t}\right)\left(\delta_{1h} + \delta_{2h}\right) + \frac{1}{8}\delta_{1h}\delta_{2h}\left(\delta_{3t} - \delta_{3h}\right)
\end{aligned}
$$

because there are 4 outcomes for the first 2 coins (honest) and all except for the outcome tt force 3 into the state heads. With $f_3(1,2,3) = f_1(1) f_1(2) f_1(3) + f_1(1) g(2,3) + f_1(2) g(1,3) + f_1(3) g(1,2) + h(1,2,3)$ we find

$$
\begin{aligned}
h(1,2,3) &= \frac{1}{8} + \frac{1}{8}\left(\delta_{3h} - \delta_{3t}\right)\left(\delta_{1h} + \delta_{2h}\right) + \frac{1}{8}\delta_{1h}\delta_{2h}\left(\delta_{3t} - \delta_{3h}\right) \\
&\quad - \frac{1}{8} - \frac{1}{8}\left[\delta_{2h}\delta_{3h} - \delta_{2h}\delta_{3t}\right] - \frac{1}{8}\left[\delta_{1h}\delta_{3h} - \delta_{1h}\delta_{3t}\right] \\
&= +\frac{1}{8}\delta_{1h}\delta_{2h}\left(\delta_{3t} - \delta_{3h}\right)
\end{aligned}
$$

The 2 point correlations for 1,3 show that there is an enhanced likelhood for an outcome hh and a reduced probability for ht (which is excluded a an outcome by the rules in (c). The same applies to $g(2,3)$. The three point correlation h actually does the opposite by increasing the likelihood of hht and reducing that of hhh. The reason for this is thta the 3 point correlations correct the 2 point correlations. The chances of betting on ttt under these rules are unchanged compared to honest coins. This is visible in the 2 point and 3 point correlations because neither changes the probability of the outcome tt (for 2 point) or ttt (for 3 point).

2. Fokker-Planck Collision Terms:

A simple approximation of the Fokker Planck collision terms is:

$$\left.\frac{\partial f}{\partial t}\right|_c = \nu_s \nabla_{\mathbf{v}} \cdot \left[\left(\mathbf{v} - \mathbf{v}_0\right) f + v_e^2 \nabla_{\mathbf{v}} f \right]$$

Compute the following zero, first, and second moments for the collision term in the fluid eqautions:

$$\begin{aligned} I_{c0} &= \int_{-\infty}^{\infty} d^3v \; \left.\frac{\partial f}{\partial t}\right|_c \\ I_{c1} &= \int_{-\infty}^{\infty} d^3v \; v_x \left.\frac{\partial f}{\partial t}\right|_c \\ I_{c2} &= \int_{-\infty}^{\infty} d^3v \; v_x^2 \left.\frac{\partial f}{\partial t}\right|_c \end{aligned}$$

For each of these integrals discuss the influence and meaning of the parameters $\mathbf{v}_0$ and v_e. Under what conditions is mass, momentum and energy conserved if the terms resulting from the above moments are used as source terms in the fluid equations.

Hint: Make use of the definitions:

$$n u_x = \int_{-\infty}^{\infty} d^3v v_x f \qquad \text{and} \qquad p_{xx} = m \int_{-\infty}^{\infty} d^3v \left(v_x - u_x\right)^2 f$$

Solution:

(1) 1st collision integral:

$$\begin{aligned} I_{c0} &= \int_{-\infty}^{\infty} d^3v \nu_s \nabla_{\mathbf{v}} \cdot \left[\left(\mathbf{v} - \mathbf{v}_0\right) f_s + v_e^2 \nabla_{\mathbf{v}} f_s \right] \\ &= \nu_s \sum_i \int_{-\infty}^{\infty} d^3v \partial_{v_i} \left[\left(v_i - v_{i0}\right) f_s + v_e^2 \partial_{v_i} f_s \right] \\ &= \nu_s \sum_i \int_{-\infty}^{\infty} d^2v \left[\left(v_i - v_{i0}\right) f_s + v_e^2 \partial_{v_i} f_s \right]_{v_i=-\infty}^{v_i=\infty} = 0 \end{aligned}$$

The result shows that no mass is generated by the collision term.

2nd collision integral:

$$\begin{aligned} I_{c1} &= \int_{-\infty}^{\infty} d^3v \nu_s v_x \nabla_{\mathbf{v}} \cdot \left[\left(\mathbf{v} - \mathbf{v}_0\right) f_s + v_e^2 \nabla_{\mathbf{v}} f_s \right] \\ &= \nu_s \int_{-\infty}^{\infty} d^3v v_x \partial_{v_x} \left[\left(v_x - v_{x0}\right) f_s + v_e^2 \partial_{v_x} f_s \right] \\ &\quad + \nu_s \sum_{i \neq x} \int_{-\infty}^{\infty} d^3v v_x \partial_{v_i} \left[\left(v_i - v_{i0}\right) f_s + v_e^2 \partial_{v_i} f_s \right] \\ &= \nu_s \int_{-\infty}^{\infty} d^3v \left\{ \partial_{v_x} v_x \left[\left(v_x - v_{x0}\right) f_s + v_e^2 \partial_{v_x} f_s \right] - \left(\partial_{v_x} v_x\right) \left[\left(v_x - v_{x0}\right) f_s + v_e^2 \partial_{v_x} f_s \right] \right\} \\ &\quad + \nu_s \sum_{i \neq x} \int_{-\infty}^{\infty} d^2v \left[v_x \left(v_i - v_{i0}\right) f_s + v_x v_e^2 \partial_{v_i} f_s \right]_{v_i=-\infty}^{v_i=\infty} \\ &= -\nu_s \int_{-\infty}^{\infty} d^3v \left[\left(v_x - v_{x0}\right) f_s + v_e^2 \partial_{v_x} f_s \right] \\ I_{c1} &= -\nu_s \int_{-\infty}^{\infty} d^3v \left[\left(v_x - v_{x0}\right) f_s \right] = -\nu_s n \left(u_x - v_{x0}\right) \end{aligned}$$

Momentum is generated if the bulk velocity u_x is different from v_{x0}. The collision term is meant to generate a momentum exchange with a second species that moves with the velocity v_{x0}.

3rd collision integral:

$$
\begin{aligned}
I_{c2} &= \int_{-\infty}^{\infty} d^3v \nu_s v_x^2 \nabla_{\mathbf{v}} \cdot \left[(\mathbf{v} - \mathbf{v}_0) f_s + v_e^2 \nabla_{\mathbf{v}} f_s \right] \\
&= \nu_s \int_{-\infty}^{\infty} d^3v v_x^2 \partial_{v_x} \left[(v_x - v_{x0}) f_s + v_e^2 \partial_{v_x} f_s \right] \\
&\quad + \nu_s \sum_{i \neq x} \int_{-\infty}^{\infty} d^3v v_x^2 \partial_{v_i} \left[(v_i - v_{i0}) f_s + v_e^2 \partial_{v_i} f_s \right] \\
&= \nu_s \int_{-\infty}^{\infty} d^3v \left\{ \partial_{v_x} v_x^2 \left[(v_x - v_{x0}) f_s + v_e^2 \partial_{v_x} f_s \right] - \left(\partial_{v_x} v_x^2 \right) \left[(v_x - v_{x0}) f_s + v_e^2 \partial_{v_x} f_s \right] \right\} \\
&\quad + \nu_s \sum_{i \neq x} \int_{-\infty}^{\infty} d^2v \left[v_x^2 (v_i - v_{i0}) f_s + v_x v_e^2 \partial_{v_i} f_s \right]_{v_i=-\infty}^{v_i=\infty} \\
&= -2\nu_s \int_{-\infty}^{\infty} d^3v \left[v_x (v_x - v_{x0}) f_s + v_x v_e^2 \partial_{v_x} f_s \right] \\
&= -2\nu_s \int_{-\infty}^{\infty} d^3v (\tilde{v}_x + u_x)(\tilde{v}_x + u_x - v_{x0}) f_s - 2\nu_s v_e^2 \int_{-\infty}^{\infty} d^3v \left[\partial_{v_x} (v_x f_s) - f_s \right] \\
&= -2\nu_s \int_{-\infty}^{\infty} d^3v \left(\tilde{v}_x^2 + u_x (u_x - v_{x0}) \right) f_s + 2\nu_s n v_e^2 \\
I_{c2} &= -2\nu_s \left(\frac{p_{xx}}{m} + n u_x (u_x - v_{x0}) \right) + 2\nu_s n v_e^2
\end{aligned}
$$

where we have used repeatedly integrations by parts und the substitution $v_x = \tilde{v}_x + u_x$. Assuming an isotropic distribution with $p_{xx} = nk_BT = mnv_{th}^2$ enders the result of the energy collision term to $I_{c2} = -2\nu_s n u_x (u_x - v_{x0}) - 2\nu_s n \left(v_{th}^2 - v_e^2 \right)$. Thus is causes heating if $u_x < v_x$ from the momentum collisions and it causes heating if $v_{th} < v_e$. Here v_e is apparently meant to be the thermal velocity of the second species.

3. Two-electron beam instability:

A plasma is described by two cold electron beams $f_{e0}(\mathbf{v}) = c_e [\delta(v_x - v_0) + \delta(v_x + v_0)] \exp\left[-\left(v_y^2 + v_z^2\right)/2u_e^2\right]$ and a neutralizing ion background of density n_0 .

(a) Show that the normalization constant satisfies $2c_e = n_0 \left(2\pi u_e^2\right)^{-1}$.

(b) Calculate the reduced distribution function (ignore ion contributions) and show that it is

$$g(v_x) = \frac{1}{n_0} \int_{-\infty}^{\infty} dv_y dv_z [f_{e0}(v)] = \frac{1}{2} [\delta(v_x - v_0) + \delta(v_x + v_0)]$$

(c) Evaluate the dielectric function

$$\epsilon = 1 + \frac{\omega_e^2}{k^2} \int du \frac{d_u g(u)}{\omega/k - u} = 0$$

and show that the dispersion relation becomes: $2 - \omega_e^2/(\omega - kv_0)^2 - \omega_e^2/(\omega + kv_0)^2 = 0$.

(d) Evaluate and discuss the dispersion relation for ω^2. What is the condition for maximum growth and what is the maximum growth rate?

General Hint: Integration over velocity space: Use substitutions such as $s_x^2 = \frac{m_s v_x^2}{2k_B T}$. Integrals:

$$\begin{aligned} \int_{-\infty}^{\infty} \exp\left(-x^2\right) dx &= \sqrt{\pi} \\ \int_{-\infty}^{\infty} x^2 \exp\left(-x^2\right) dx &= \frac{\sqrt{\pi}}{2} \end{aligned}$$

Solution:

(a) Normalization:

$$\begin{aligned} n_0 &= \int dv_x dv_y dv_z f_{e0} \\ &= c_e \int dv_x dv_y dv_z [\delta(v_x - v_0) + \delta(v_x + v_0)] \exp\left[-\left(v_y^2 + v_z^2\right)/2u_e^2\right] \\ &= 2c_e \int dv_y dv_z \exp\left[-\left(v_y^2 + v_z^2\right)/2u_e^2\right] \\ &= 2c_e \left(2u_e^2\right) \int d\tilde{v}_y d\tilde{v}_z \exp\left[-\left(\tilde{v}_y^2 + \tilde{v}_z^2\right)\right] = 2c_e \left(2\pi u_e^2\right) \end{aligned}$$

(b) Reduced distribution function:

$$\begin{aligned} g(v_x) &= \frac{1}{n_0} \left[\int\int dv_y dv_z f_{e0}\right] \\ &= c_e \frac{(2\pi u_e^2)[\delta(v_x - v_0) + \delta(v_x + v_0)]}{n_0} \\ &= \frac{1}{2} [\delta(v_x - v_0) + \delta(v_x + v_0)] \end{aligned}$$

(c) Dielectric function:

$$\begin{aligned} \epsilon &= 1 + \frac{\omega_e^2}{k^2} \int du \frac{d_u g(u)}{\omega/k - u} \\ &= 1 - \frac{\omega_e^2}{k^2} \int du \frac{g(u)}{(\omega/k - u)^2} \end{aligned}$$

$$
\begin{aligned}
&= 1-\frac{\omega_e^2}{2k^2}\int du\frac{\delta\left(u-v_0\right)+\delta\left(u+v_0\right)}{\left(\omega/k-u\right)^2}\\
&= 1-\frac{\omega_e^2}{2k^2}\left[\frac{1}{\left(\omega/k-v_0\right)^2}+\frac{1}{\left(\omega/k+v_0\right)^2}\right]\\
\epsilon &= 1-\frac{\omega_e^2}{2}\left[\frac{1}{\left(\omega-kv_0\right)^2}+\frac{1}{\left(\omega+kv_0\right)^2}\right]=0
\end{aligned}
$$

(d) Evaluation of the dispersion relation:

$$
\begin{aligned}
\left(\omega^2-k^2v_0^2\right)^2-\frac{1}{2}\omega_e^2\left[\left(\omega+kv_0\right)^2+\left(\omega-kv_0\right)^2\right] &= 0\\
\left(\omega^2-k^2v_0^2\right)^2-\omega_e^2\left[\omega^2+k^2v_0^2\right] &= 0\\
\omega^4-2\left(k^2v_0^2+\frac{1}{2}\omega_e^2\right)\omega^2+\left(k^2v_0^2+\frac{1}{2}\omega_e^2\right)^2 &= \left(k^2v_0^2+\frac{1}{2}\omega_e^2\right)^2-k^4v_0^4+\omega_e^2k^2v_0^2\\
&= 2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4\\
\omega^2 &= k^2v_0^2+\frac{1}{2}\omega_e^2\pm\left(2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4\right)^{1/2} \qquad (1)
\end{aligned}
$$

Consider '$-$' sign to obtain instability for $\omega^2<0$, i.e, if sqrt term dominates:

$$
\begin{aligned}
2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4 &> \left(k^2v_0^2+\frac{1}{2}\omega_e^2\right)^2=k^4v_0^4+k^2v_0^2\omega_e^2+\frac{1}{4}\omega_e^2\\
\omega_e^2 &> k^2v_0^2
\end{aligned}
$$

i.e., for $k^2<\omega_e^2/v_0^2$ the system is unstable.
Obtain a minimum of

$$
\omega^2=I\left(k\right)=k^2v_0^2+\frac{1}{2}\omega_e^2-\left(2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4\right)^{1/2}
$$

for $k^2<\omega_e^2/v_0^2$. An instability requires that the minimum of ω^2 is negative.

$$
\begin{aligned}
I' &= 2kv_0^2-2\omega_e^2kv_0^2\left(2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4\right)^{-1/2}=0\\
\left(2\omega_e^2k^2v_0^2+\frac{1}{4}\omega_e^4\right) &= \omega_e^4\\
k^2v_0^2 &= \frac{3}{8}\omega_e^2
\end{aligned}
$$

and

$$
\begin{aligned}
\omega^2 &= \frac{3}{8}\omega_e^2+\frac{1}{2}\omega_e^2-\left(\frac{3}{4}\omega_e^4+\frac{1}{4}\omega_e^4\right)^{1/2}\\
&= -\frac{1}{8}\omega_e^2\\
\omega &= \pm i\frac{\omega_e}{\sqrt{8}}
\end{aligned}
$$

Summary: For $k=0$ we find $\omega=0$; for increasing $|k|$ -> imaginary ω with a maximum absolute value of $\omega_e/\sqrt{8}$ at $k^2v_0^2=\frac{3}{8}\omega_e^2$. For larger k the growth rate decreases and passes through 0 for $k^2v_0^2=\omega_e^2$. For values of k larger than this the solution is a non-growing wave with the k dependence from (1).

4. Harris sheet:

The distribution functions for the Harris sheet (for invariance along y or $\partial/\partial_y = 0$) is given by

$$f_s(\mathbf{v},\mathbf{x}) = n_0\left(\frac{m_s}{2\pi k_B T}\right)^{3/2}\exp\left[\frac{q_s u_s}{k_B T}A(\mathbf{x})\right]\exp\left[-\frac{m_s}{2k_B T}\left(v_x^2+v_z^2+(v_y-u_s)^2\right)\right]$$

with the vector potential $\mathbf{A} = A\mathbf{e}_y$ (using exact neutrality and $u_e = -u_i = u$).

(a) Show by explicit substitution that f_s solves the collisionless Boltzmann equation.

(b) Compute pressure and current density and demonstrate $j_y(A) = dp(A)/dA$

Solution:

(a) Collisionless Boltzmann equation:

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\nabla f_s + \frac{q_s}{m_s}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}} f_s = 0$$

Here we may need the electric and magnetic fields :

$$\begin{aligned}\mathbf{E} &= -\frac{\partial A}{\partial t}\mathbf{e}_y = 0\\ \mathbf{B} &= \nabla\times(A\mathbf{e}_y) = \partial_x A\mathbf{e}_{\mathbf{z}}\end{aligned}$$

Terms in the Boltzmann equation: 1st term (trivial):

$$\frac{\partial f_s}{\partial t} = 0$$

2nd term (since there is only an x dependence through $A(x)$):

$$\begin{aligned}T_2 = \mathbf{v}\cdot\nabla f_s &= \frac{q_s u_s}{k_B T} f_s(x,\mathbf{v})\, v_x \partial_x A(\mathbf{x})\\ &= \frac{q_s u_s}{k_B T} v_x B_z f_s(x,\mathbf{v})\end{aligned}$$

3rd term:

$$\begin{aligned}T_3 &= \frac{q_s}{m_s}\mathbf{v}\times\mathbf{B}\cdot\nabla_{\mathbf{v}} f_s = \frac{q_s}{m_s}\mathbf{v}\times(\partial_x A\mathbf{e}_{\mathbf{z}})\cdot\nabla_{\mathbf{v}} f_s\\ &= -\frac{q_s}{m_s}\frac{m_s}{k_B T} f_s\left[\mathbf{v}\times(\partial_x A\mathbf{e}_{\mathbf{z}})\right]\cdot(\mathbf{v}-u_s\mathbf{e}_y)\\ &= -\frac{q_s}{m_s}\frac{m_s}{k_B T}\left[u_s v_x \partial_x A\right] f_s\\ &= -q_s\frac{u_s}{k_B T} v_x B_z f_s\end{aligned}$$

Alternatively you can expicitely use the magnetic field and find for the term in square brackets $v_y B_z v_x - v_x B_z (v_y - u_s) = v_x B_z u_s$ which is also the same as in term 2.

This demonstrates that the 2nd and 3rd term contributions cancel.

(b) Pressure:

$$\begin{aligned}p_s &= \frac{m_s}{3}\int\left(v_x^2+(v_y-u_s)^2+v_z^2\right)F_s(H_s,P_{sy})\,d^3v\\ j_{sy} &= q_s\int v_y F_s(H_s,P_{sy})\,d^3v\end{aligned}$$

Here we have only a current in the $y$direction because of symmetry. With the transformation $\tilde{v}_y = v_y - u_s$ the current density is therefore

$$\begin{aligned} j_{sy} &= q_s \int (\tilde{v}_y + u_s) f_s d^3v = q_s u_s \int f_s d^3v \\ &= q_s n u_s n \exp\left[\frac{q_s u_s}{k_B T} A(\mathbf{x})\right] \end{aligned}$$

For the pressure we obtain:

$$\begin{aligned} \frac{\partial p_s}{\partial A} &= \frac{m_s}{3}\frac{\partial}{\partial A}\int \left(v_x^2 + (v_y - u_s)^2 + v_z^2\right) f_s d^3v \\ &= m_s \int v_x^2 \frac{\partial}{\partial A} f_s d^3v \\ &= m_s \frac{q_s u_s}{k_B T}\int v_x^2 f_s d^3v \\ &= m_s \frac{q_s u_s}{k_B T}\frac{n k_B T}{m_s}\exp\left[\frac{qu}{k_B T} A(\mathbf{x})\right] \\ \frac{\partial p_s}{\partial A} &= q_s n u_s n \exp\left[\frac{q_s u_s}{k_B T} A(\mathbf{x})\right] = j_{sy} \end{aligned}$$