1. Anisotropic distribution function: Consider a magnetic field along the z direction and a distribution function

$$f(\mathbf{v}) = c_0 \exp\left(-\frac{mv_z^2}{2k_B T_\parallel} - \frac{m\left(v_x^2+v_y^2\right)}{2k_B T_\perp}\right)$$

a) Show that normalization of the density to n_0 yields

$$c_0 = n_0 \left(\frac{m}{2\pi k_B T_\parallel}\right)^{1/2} \left(\frac{m}{2\pi k_B T_\perp}\right)$$

b) Compute the average energy parallel and perpendicular to the magnetic field.

2. Field line equation: Consider the magnetic field $B_x = B_0 y/L$, $B_y = \epsilon B_0 x/L$.

a) Compute the z component of the vectorpotential
b) Determine the equation for magnetic field lines.
c) Discuss and plot the magnetic field lines.
d) Compute the current density. Is there a value for ϵ for which this magnetic field is a vacuum field?

3. Klimontovich Equation: Starting from the Klimontovich distribution, derive the Klimontovich equation. What procedure and conditions are necessary to obtain the collisionless Boltzmann equation?

4. Harris sheet: The distribution functions for the exact neutral Harris sheet equilibrium are given by

$$\begin{aligned} f_s(\mathbf{v},\mathbf{r}) &= c_s \exp\left(\frac{q_s u_s}{k_B T} A_y(x,z)\right) \exp\left(-\frac{m_s}{2k_B T}\left(v_x^2+v_z^2+(v_y-u_s)^2\right)\right) \\ \text{with} \quad & c_s = n_0 \left(\frac{m_s}{2\pi k_B T}\right)^{3/2} \\ \text{and} \quad & u_e = -u_i = u_0 \end{aligned}$$

(a) show that the associated current density is: $j_y(A_y) = -2en_0u_0 \exp\left(-\frac{eu_0}{k_B T} A_y(\mathbf{r})\right)$
(b) Show that the pressure $p = p_e + p_i$ is: $p(A_y) = 2n_0 k_B T \exp\left(-\frac{eu_0}{k_B T} A_y(\mathbf{r})\right)$
(c) Show by explicit substitution that this distribution satisfies the collisionless Boltzmann with $\partial/\partial t = 0$ and $\partial/\partial y = 0$.

Hint: Integrate the distributions over velocity space. Use substitutions such as $s_x^2 = \frac{m_s v_x^2}{2k_B T}$. Integrals:

$$\begin{aligned} \int_{-\infty}^{\infty} \exp\left(-x^2\right) dx &= \sqrt{\pi} \\ \int_{-\infty}^{\infty} x^2 \exp\left(-x^2\right) dx &= \frac{\sqrt{\pi}}{2} \end{aligned}$$

This midterm will be graded like the final exam if you wish so but it is for your own evaluation only and the results will not be counted. You need to select and solve 3 of the 4 problems only for the full score. If you want a proper evaluation keep in mind that the final exam is 3 hours and open book (any plasma physics book, and plasma or math formulary but not open homework).