1. Maxwellian Integrals:

Plasma physics makes frequent use integrals of integrals over the Maxwell distribution.
(a) Show

$$\int_{-\infty}^{\infty} \exp\left(-x^2\right) dx = \sqrt{\pi}$$

(b) Prove that

$$\int_0^{\infty} x^m \exp\left(-cx^2\right) dx = \begin{cases} \Gamma\left(l+1/2\right)/2c^{l+1/2} & \text{for} \quad m = 2l \\ l!/2c^{l+1} & \text{for} \quad m = 2l+1 \end{cases}$$

for m integer and > 0. Hint: $\Gamma\left(1/2\right) = \sqrt{\pi}$ and use recursion.

2. Magnetic gradient drift:

Evaluate the time average for the gradient drift velocity

$$\mathbf{v}_{\nabla} = \frac{1}{B_0}\left\langle \mathbf{v}_g x \frac{dB_0}{dx} \right\rangle$$

by using the general motion of a single particle with charge q and mass m in a magnetic field along the z direction with a gradient in the the x direction (without any electric field in the Lorentz equation). Show that the resulting drift is in the y direction and can be written as

$$\mathbf{v}_{\nabla} = \frac{mv_{\perp}^2}{2qB^3}\mathbf{B} \times \nabla B$$

3. Drift currents:

Assume a plasma consisting of electrons and ions with equal number density n and temperature T. If electrons and ions have (in the average) different drift velocities, the relative particle drift sets up a current. Determine this current for
(a) the electric field ($\mathbf{E} \times \mathbf{B}$) drift,
(b) the gradient B drift,
(c) the curvature B drift, and
(d) the polarization drift,
using the expressions derived in section 1.4.1. In order to express the average parallel and perpendicular particle velocities (squared) for the gradient B and curvature drifts assume that the particles are in thermal equilibrium, i.e., Maxwell distributed. **Note** that the drift expressions are derived for individual particles, however, the current density is a property of the total particle distribution.

Please turn in the solutions to the homework on Tuesdayday, 2/07/2012