4. Klimontovich equation:

The derivation of the Klimontovich equation requires (a) to replace the particle velocities $\mathbf{V}_i = d\mathbf{R}_i/dt$ with phase space velocity coordinates

$$\dot{\mathbf{R}}_i(t) \cdot \nabla_{\mathbf{r}} \delta(\mathbf{r} - \mathbf{R}_i(t))\, \delta(\mathbf{v} - \mathbf{V}_i(t)) = \mathbf{v} \cdot \nabla_{\mathbf{r}} \delta(\mathbf{r} - \mathbf{R}_i(t))\, \delta(\mathbf{v} - \mathbf{V}_i(t))$$

and (b) to replace the particle acceleration $d\mathbf{V}_i/dt$ with the Lorentz Force term at phase space coordinates $\mathbf{r}, \mathbf{v}$, i.e.,

$$\begin{aligned} d\mathbf{V}_i/dt &= \frac{q}{m}\left\{\mathbf{E}(\mathbf{R}_i(t), t) + \mathbf{V}_i(t) \times \mathbf{B}(\mathbf{R}_i(t), t)\right\} \cdot \nabla_{\mathbf{v}} \delta(\mathbf{r} - \mathbf{R}_i(t))\, \delta(\mathbf{v} - \mathbf{V}_i(t)) \\ &= \frac{q}{m}\left\{\mathbf{E}(\mathbf{r}, t) + \mathbf{v} \times \mathbf{B}(\mathbf{r}, t)\right\} \cdot \nabla_{\mathbf{v}} \delta(\mathbf{r} - \mathbf{R}_i(t))\, \delta(\mathbf{v} - \mathbf{V}_i(t)) \end{aligned}$$

Demonstrate rigorously that this is justified for (a) and (b).

5. Liouville Equation:

Prove the equivalence between the convective form of the Liouville equation ($DN/Dt = 0$) and the continuity equation ($\partial N/\partial t + \sum_{i=1}^{N_0} \nabla_{\mathbf{r}_i} \cdot (\mathbf{v}_i N) + \sum_{i=1}^{N_0} \nabla_{\mathbf{v}_i} \cdot \left(\dot{\mathbf{V}}_i N\right) = 0$).

6. BBGKY Hierarchy:

Demonstrate that the recurrence relation

$$\begin{aligned} \frac{\partial f_k}{\partial t} + \sum_{i=1}^{k} \mathbf{v}_i \cdot \nabla_{\mathbf{r}_i} f_k + \sum_{i=1}^{k} \sum_{j=1}^{k} \mathbf{a}_{ij} \cdot \nabla_{\mathbf{v}_i} f_k & \\ + \frac{N_0 - k}{V} \sum_{i=1}^{k} \int_{-\infty}^{\infty} d\mathbf{r}_{k+1} d\mathbf{v}_{k+1} \mathbf{a}_{i,k+1} \cdot \nabla_{\mathbf{v}_i} f_{k+1} &= 0 \end{aligned} \tag{1}$$

for the equations in the BBGKY hierarchy is correct. You can do this by induction, i.e., take the equation for f_k and integrate it with respect to $d\mathbf{r}_k d\mathbf{v}_k$ and show that the resulting equation for f_{k-1} is consistent with the general form for equation for f_k.

Please turn in the solutions to the homework on Tuesday day, 2/14/2012