7. Three-Point correlation (Coins):

In deriving the equation for f_1 and $g(1,2,t)$ we defined the joint probability f_3 in terms of the one-point probability f_1, the two-point correlation function g, and the three-point correlation function h. Suppose we apply this to the case of three coins, each of which can come up with heads (+) or tails (-). What is the meaning of f_3 in this case? Write out f_3 in the form

$$f_3(1,2,3) = f_1(1)f_1(2)f_1(3) + f1(1)g(2,3) + f1(2)g(1,3) + f1(3)g(1,2) + h(1,2,3),$$

and evaluate f_3, f_1, g, and h in each of the following cases.
(a) All three coins are "honest", that is, each coin is equally likely to come up heads or tails, and each coin is unaffected by any other coin.
(b) Because the coins are mysteriously locked together, in any one throw all three are heads or tails, the result changing randomly from throw to throw.
(c) The first two coins always come up heads, while the third is honest. Note that here the probability functions are not symmetric, so that for example, $f_1(1)$ is not the same as $f_1(3)$.

(d) The first coins two are honest but 3 is manipulated to show heads if 1 and 2 are heads. How is your chance improving if you bet on +++ and can you see this looking at the two- or thee-point correlation?

8. Fourier Transform:

Show that the three-dimensional Fourier transforms of the electrostatic potential energy $\varphi(\mathbf{r}) = e^2/(4\pi\epsilon_0 |\mathbf{r}|)$ and of the electrostatic force $\mathbf{a_{12}}(\mathbf{r}) = e^2\mathbf{r}/\left(4\pi\epsilon_0 m_e |\mathbf{r}|^3\right)$ are $\tilde{\varphi}(\mathbf{k}) = e^2/(\epsilon_0 (2\pi)^{3/2} |\mathbf{k}|^2)$ and $\mathbf{a_{12}}(\mathbf{k}) = -i\mathbf{k}\tilde{\varphi}(\mathbf{k})/m_e$ respectively. (Hint: To obtain the transform for the potential a derive the result considering first the transform of $\varphi'(\mathbf{r}) = \varphi(\mathbf{r})\exp(-\nu r)$ with $\nu > 0$ and after you obtained the result take the limit of $\nu \to 0$).

9. Lenard Balescu Equation:

Demonstrate that a Maxwellian ($f(\mathbf{v}) = const \cdot \exp(mv^2/2T)$) is an exact time independent solution of the Lenard Balescu (LB) equation.

Please turn in the solutions to the homework on Tuesday day, 2/28/2012