10. Ion-acoustic Waves:

The dispersion relation

$$1 - \frac{\omega_{pe}^2}{k^2} \int_{-\infty}^{\infty} du \frac{d_u g\,(u)}{u - \omega/k} = 0$$

has a branch describing ion-acoustic waves as well as one describing Langmuir waves. Consider $g(u)$ with electron and ion Maxwellian contributions as introduced in class with $T_i \ll T_e$ and look for a wave with a phase speed such that

$$v_i \ll \omega/k \ll v_e$$

(v_i, v_e are ion and electron thermal velocities). Expand the denominator (for the ion contribution) in the above relation as we did for the Langmuir waves. For the electron contribution approximate the integral by assuming $\omega/k \ll u$ in the denominator. Then solve the dispersion relation $\epsilon\,(k, \omega) = 0$ to find $\omega\,(k)$ for ion-acoustic waves.

11. Contour Integration:

(a) Show that the integral $\oint 1/z \; dz$ along a closed contour around the origin yields the result $2\pi i$ (choose a closed circle with radius R and represent z in terms of the cylindrical coordinates r and φ.).
(b) Prove the residue theorem by generalizing the result from (a) to the case $\oint f(z)/z \; dz$ and assume the $f(z)$ is analytic, i.e., that you can expand it in a power series of z in the vicinity of the origin. How can it be generalized to the case of a simple pole at $z = a$ instead of the origin?
(c) Find the principal value of $\int_{-\infty}^{\infty} \cos(mx)/(x-a)\; dx$ with a real, $m > 0$.

12. Ion Acoustic Wave Properties and Damping:

The real part of the dielectric function for ion acoustic waves is

$$\epsilon_r = 1 + \frac{1}{k^2 \lambda_e^2} - \frac{m_e \omega_e^2}{m_i \omega^2} \left(1 + 3 \frac{v_i^2 k^2}{\omega^2} \right)$$

with $\lambda_e =$ electron Debye length, $\omega_e =$ electron plasma frequency, and $v_i = \sqrt{k_B T_i / m_i} =$ ion thermal velocity. Determine the damping rate for Landau damping. Discuss the dispersion relation (ω_r and ω_i) as a function of wave number under the constraints $\omega/k \ll v_e$ and $\omega/k \gg v_i$. For what conditions (wave number) becomes Landau damping dominant. Is this condition consistent with the constraints?

Please turn in the solutions to the homework on Tuesday day, 3/6/2012