13. Electrostatic Dispersion relation:

The dispersion relation for electrostatic waves

$$1 + \frac{\omega_e^2}{k^2} \int du \frac{d_u g(u)}{\omega/k - u} = 0$$

must be solved using the Landau contour; alternatively, the integral can be evaluated for $\omega_i > 0$ and the result analytically continued to $\omega_i < 0$. Evaluate the dispersion relation and find the normal modes $\omega(k)$ for the following distributions $g(u)$.
(a) Cold plasma, $g(u) = \delta(u)$.
(b) Cold beam, $g(u) = \delta(u - u_0)$.
(c) Square distribution, $g(u) = (2c)^{-1}$ for $|u| < c$, $g(u) = 0$ for $|u| > c$, with c real and positive.
(d) Cauchy distribution, $g(u) = (c/\pi)(u^2 + c^2)^{-1}$.

14. Kappa Distribution:

A Kappa distribution function is defined by

$$f_\kappa(\mathbf{v}) = c_N[1 + \frac{m\mathbf{v}^2}{2\kappa W_0}]^{-(\kappa+1)}$$

with the normalization constant c_N and the typical energy W_0.
(a) Describe qualitatively the properties of this distribution function in comparison with a Maxwellian. Why is it necessary to assume $\kappa > 1$?
(b) Determine the normalization constant c_N for $\kappa = 2$ if the plasma density is given by n_0 for one species.
(c) Determine the average kinetic energy for $\kappa = 2$.

15. Penrose Criterion:

Consider the case of a reduced distribution function $g(u)$ with 2 maxima at $u = u_0$ and $u = u_2$, and a minimum at $u = u_1$.
(a) Show that if $g(u_1) = 0$, the Penrose criterion for instability is always satisfied.
(b) Suppose the lower maximum of g is at $u = u_2$. Following the arguments given in class, show that the range of unstable waves is given by

$$\omega_e^2 \int_{-\infty}^{\infty} du \frac{[g(u) - g(u_2)]}{(u - u_2)^2} < k^2 < \omega_e^2 \int_{-\infty}^{\infty} du \frac{[g(u) - g(u_1)]}{(u - u_1)^2}.$$

Why is the the lower limit of k^2 in the above expression chosen with the integral at u_2 rather than by the equivalent integral for u_0?

Please turn in the solutions to the homework on Thursday day, 3/22/2012