15. Anisotropic distribution function

An anisotropic distribution function for a two-dimensional equilibrium ($\partial/\partial y = 0$) can be written as $F(H, \mu) = G(A) \exp(-\beta(A)H - \eta(A)\mu B_0)$ with the Hamiltonian $H = (m/2)\left(v_\perp^2 + v_\parallel^2\right) + q\phi$, the adiabatic moment $\mu = (m/2)v_\perp^2/B$, the magnetic field strength B, and the y component of the vector potential A, where $G(A)$, $\beta(A)$, and $\eta(A)$ are arbitrary functions.

(a) Relate β and η to the temperatures $T_\parallel$ and $T_\perp$ for a bi-Maxwellian distribution.

(b) Calculate the number density n by integrating F over velocity space.

(c) Show that the parallel and perpendicular pressures are $p_\parallel = n(A, B)k_B T_\parallel$ and $p_\perp = n(A, B)k_B T_\perp$

(d) Determine the pressure anisotropy $D = \frac{p_\parallel - p_\perp}{p_\parallel}$ as a function of B, B_0, β, and η.

16. Magnetotail equilibrium

An asymptotic two-dimensional magnetotail equilibrium is determined by the y component of the vector potential

$$A(x_1, z) = A_0 \ln\left[l(x_1)\cosh\left(z/l(x_1)\right)\right] \qquad with \qquad l(x_1) = \left(2p_0(x_1)\right)^{-1/2}$$

with $x_1 = \varepsilon x$ and $\varepsilon \ll 1$.

(a) Determine the magnetic field components, the current density, and the pressure resulting from this choice of the vector potential. Describe the magnetic field configuration.

(b) Examine the force balance equation and determine the deviation from exact force balance along the x and z direction. What are the conditions that this error remains small?

16. Project:

Provide a two page report on your progress with the class project.

Please turn in the solutions to the homework on Thursday day, 4/13/2012