21. Stability of the Z pinch: The pressure and magnetic field profile of the Z pinch is

$$B_\theta = \frac{\mu_0 j_0}{2} r \qquad and \qquad p = -\frac{\mu_0 j_0^2}{4} r^2 + p_0 \qquad for \qquad r \le r_c = \left(\frac{4p_0}{\mu_0 j_0^2}\right)^{1/2}$$

Assume perturbations with $\partial/\partial\theta = 0$ and apply the stability criterion derived in class to the equilibrium to determine it equilibrium properties. How do these change if a constant pressure p_c is superimposed.

22. Stability of the Harris Sheet: To determine the stability criterion for the Harris sheet equilibrium $\mathbf{B} = B\mathbf{e}_y = B_0 \tanh\frac{x}{L}\,\mathbf{e}_y$ use the Potential

$$\begin{aligned} U &= \frac{1}{2}\int_V \Big[\gamma p_0 \left(\nabla\cdot\boldsymbol{\xi}\right)^2 + \frac{1}{\mu_0}\left(\nabla\times\left(\boldsymbol{\xi}\times\mathbf{B}_0\right)\right)^2 \\ &\qquad +\boldsymbol{\xi}\cdot\nabla p_0 \nabla\cdot\boldsymbol{\xi} - \frac{1}{\mu_0}\left(\boldsymbol{\xi}\times\left(\nabla\times\mathbf{B}_0\right)\right)\cdot\nabla\times\left(\boldsymbol{\xi}\times\mathbf{B}_0\right)\Big]\, d\mathbf{x} \end{aligned}$$

for $\partial\boldsymbol{\xi}/\partial y = 0$. (Perturbations at the boundary at $x = \infty$ are 0.)

(a) Show that

$$\begin{aligned} \nabla\times\left(\boldsymbol{\xi}\times\mathbf{B}_0\right) &= B\left[\frac{\partial\xi_x}{\partial y}\mathbf{e}_x - \left(\frac{\partial\xi_z}{\partial z} + \frac{1}{B}\frac{\partial\xi_x B}{\partial x}\right)\mathbf{e}_y + \frac{\partial\xi_z}{\partial y}\mathbf{e}_z\right] \\ \boldsymbol{\xi}\times\left(\nabla\times\mathbf{B}_0\right) &= \frac{\partial B}{\partial x}\left(\xi_y\mathbf{e}_x - \xi_x\mathbf{e}_y\right) \end{aligned}$$

(b) Bring U into the form

$$U = \frac{1}{2}\int_V \left[a_{11}\left(\nabla\cdot\boldsymbol{\xi}\right)^2 + 2a_{12}\xi_x\nabla\cdot\boldsymbol{\xi} + a_{22}\xi_x^2\right] d\mathbf{x}$$

and show that

$$\begin{aligned} a_{11} &= \gamma p + \frac{B^2}{\mu_0} \\ a_{12} &= \frac{dp}{dx} + \frac{B}{\mu_0}\frac{dB}{dx} \\ a_{22} &= \frac{1}{\mu_0}\left(\frac{dB}{dx}\right)^2 + \frac{dp}{dx}\frac{1}{B}\frac{dB}{dx} \end{aligned}$$

(c) Evaluate the coefficients for the Harris sheet and determine and discuss the stability properties for the assumed perturbation.

23. Project:

Provide a report on your progress with the class project. Summarize the remaining tasks that you expect before you can actually present the project.

Please turn in the solutions to the homework on Thursday day, 4/26/2012