Appendix A

Appendix

A.1 Tables of Plasma Parameters

The first table gives some fundamental constants:

Property	Symbol	SI	cgs
Speed of light	c	3×10^{8} m s^{-1}	3×10^{10} m s^{-1}
Boltzmann constant	k	1.38×10^{-23} J K^{-1}	1.38 erg K^{-1}
Electron mass	m_e	9.1 $\times10^{-31}$ kg	9.1 $\times10^{-28}$ g
Proton mass	m_i	1836 m_e	1836 m_e
Elementary charge	e	1.6×10^{-19}C	4.8×10^{-10} statcoulomb
Dielectric constant	ϵ_0	8.85×10^{-12} F m^{-1}	-
Permeability of free space	μ_0	$4\pi\times10^{-7}$ H m^{-1}	-

Other relations:

$$\begin{aligned} \epsilon_0\mu_0 &= \frac{1}{c^2} \\ 1eV &= 1.16\times 10^4 K \end{aligned}$$

The following table presents several basic plasma frequencies, length scales and velocities:

Property	Symbol	SI	cgs
Plasma frequency	ω_{pe}	$\left(\frac{n_e e^2}{\epsilon_0 m_e}\right)^{1/2}$	$\left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$
Electron gyro frequency	ω_{ge}	$\frac{eB}{m_e}$	$\frac{eB}{m_e c}$
Coulomb collision frequency	ν_{ei}	$\frac{\omega_{pe}}{4\pi n\lambda_D^3}\ln\Lambda$	$\frac{\omega_{pe}}{4\pi n\lambda_D^3}\ln\Lambda$
Debye length	λ_D	$\left(\frac{\epsilon_0 kT}{ne^2}\right)^{1/2}$	$\left(\frac{kT}{4\pi ne^2}\right)^{1/2}$
Skin depth (electron inertia)	λ_e	$\frac{c}{\omega_{pe}}$	$\frac{c}{\omega_{pe}}$
Electron Gyroradius	r_{ge}	$\frac{v_{the}}{\omega_{pe}}$	$\frac{v_{the}}{\omega_{pe}}$
Electron Thermal velocity	v_{the}	$\left(\frac{kT}{m_e}\right)^{1/2}$	$\left(\frac{kT}{m_e}\right)^{1/2}$
Alfven speed	v_A	$\frac{B}{(\mu_0 n m_i)^{1/2}}$	$\frac{B}{(4\pi n m_i)^{1/2}}$
Number of particles in a Debye sphere	$N_D = \frac{1}{g}$	$\frac{4\pi}{3}n\lambda_D^3$	$\frac{4\pi}{3}n\lambda_D^3$

The corresponding ion properties are

$$\begin{aligned}
\omega_{pi} &= \sqrt{m_e/m_i}\omega_{pe} \\
\omega_{gi} &= (m_e/m_i)\omega_{ge} \\
\nu_{ie} &= (m_e/m_i)\nu_{ei} \\
r_{gi} &= \sqrt{m_i/m_e}r_{ge} \\
v_{thi} &= \sqrt{m_i/m_e}v_{the}
\end{aligned}$$

The following list provides numerical values for the various plasma parameters in a convenient form. Because it is more common in the field of magnetospheric physics everything is measured in cgs units in this table, i.e., n in cm^{-3}, B in Gauss, T in eV, and $\ln\Lambda \approx 20$. Note that 1 T = 10^4 Gauss and 1 nT =10^{-5} Gauss.

$$\begin{aligned}
\omega_{pe} &= 5.64 \times 10^4\, n^{1/2} \;\; [rad/sec] \\
\omega_{ge} &= 1.76 \times 10^7\, B \;\; [rad/sec] \\
\omega_{gi} &= 9.58 \times 10^3\, B \;\; [rad/sec] \\
\nu_{ei} &= 1.1 \times 10^{-5}\; \ln(\Lambda) n T_e^{3/2} \;\; \left[sec^{-1}\right] \\
\lambda_{De} &= 7.43 \times 10^2\, n^{-1/2} T_e^{1/2} \;\; [cm] \\
\lambda_e &= 5.31 \times 10^5\, n^{-1/2} \;\; [cm] \\
\lambda_i &= 2.28 \times 10^7\, n^{-1/2} \;\; [cm] \\
r_{ge} &= 2.38 \times 10^0\, T_e^{1/2} B^{-1} \;\; [cm] \\
r_{gi} &= 1.02 \times 10^2\, T_i^{1/2} B^{-1} \;\; [cm] \\
v_{the} &= 4.19 \times 10^7\, T_e^{1/2} \;\; [cm/sec] \\
v_{thi} &= 9.79 \times 10^5\, T_i^{1/2} \;\; [cm/sec] \\
v_A &= 2.18 \times 10^{11}\, B n^{-1/2} \;\; [cm/sec] \\
N_D &= 1.72 \times 10^9 T_e^{3/2} n^{-1/2}
\end{aligned}$$

The last table is an overview of typical plasma properties in the magnetosheath/mantle, the outer magnetosphere/tail, and the inner magnetosphere(acceleration) regions.

	Mantle/Sheath	Outer M'Sphere/Tail	Inner M'Sphere/Accel. R
n [cm^{-3}]	10 [1-100]	1 [0.01-5]	10^2 [0.1-10^4]
B [nT]	20 [5-100]	40 [10-100]	10^4 [10^3 -10^5]
T_e [eV]	50 [10-10^3]	500 [$10^2 - 10^3$]	10^3 [1-10^4]
T_i [eV]	100 [10-10^3]	2×10^3 [$10^3 - 10^4$]	10^3 [1-10^4]
ω_{pe}	1.8× 10^5	5.7× 10^4	5.6× 10^5
ω_{ge}	3.5× 10^3	7.0× 10^3	1.8× 10^6
ω_{gi}	1.9× 10^0	3.6× 10^0	9.6× 10^2
ν_{ei}	6.2× 10^{-6}	2.0× 10^{-8}	7.0× 10^{-7}
λ_{De} [m]	1.7× 10^1	1.7× 10^2	2.3× 10^1
λ_e [m]	1.7× 10^3	5.3× 10^3	5.3× 10^2
λ_i [m]	7.3× 10^4	2.3× 10^5	2.3× 10^4
r_{ge} [m]	8.4× 10^2	1.3× 10^3	7.5× 10^0
r_{gi} [m]	5.0× 10^4	1.1× 10^5	3.2× 10^2
v_{the} [m/sec]	3.0× 10^6	9.4× 10^6	1.3× 10^7
v_{thi} [m/sec]	9.8× 10^4	4.4× 10^5	3.1× 10^5
v_A [m/sec]	1.4× 10^5	8.7× 10^5	2.18× 10^7
N_D	1.9× 10^{11}	1.9× 10^{13}	5.4× 10^{12}

A.2 Definition of the radius of curvature

Assume: $x(t)$, $y(t)$, and $z(t)$

the line element along the curve $r(t) = (x(t), y(t), z(t))$ is

$$ds = dt\sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2} \tag{A.1}$$

Arc length:

$$s = \int_{t_0}^{t} ds$$

Tangent:

$$\mathbf{e}_t = \frac{d\mathbf{r}}{ds} \tag{A.2}$$

The unit vector $\mathbf{e}_n$ defined by

$$\frac{d^2\mathbf{r}}{ds^2}\frac{1}{r_c}\mathbf{e}_n \tag{A.3}$$

is the principal normal with the radius of curvature defined by

$$r_c = \sqrt{\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2} \tag{A.4}$$

Exercise: Use the field line equation for the dipole field to compute the radius of curvature from $x = r(\theta)\sin\theta$ and $x = r(\theta)\cos\theta$ as a function of θ.