Chapter 3

Electromagnetic Fields in Space

Magnetic and electric field are fundamental properties in the entire universe. Massive magnetic fields exist in the vicinity of pulsars, in active galactic nuclei, and probably around black holes. In the solar system many planets have an intrinsic magnetic field and in most cases the dipole component is the dominant contribution in particular to the far magnetic field. The sun is an exception in that the dipole component is usually small compared to locally generated fields.

	Distance (AU)	Radius R_P (10^3km)	Magn. Moment (ME)	Tilt	Rot. Tilt	Polarity	Spin (days)	MP Dist. (R_P)
Sun	0	700	4.4×10^6	var	7.25°	N	27d	-
Mercury	0.4	2.49	5.6×10^{-4}	10°	~0°	N	58.6d	1.6
Earth	1.0	6.37	1	11.5°	23.4°	N	24h	11
Mars	1.5	3.38	2×10^{-4}	<20°	6.7°	R	24.5h	1.4
Jupiter	5.2	71.4	20,000	10°	3.1°	R	10h	50
Saturn	9.5	60.4	580	1°	26.7°	N	10.65h	21
Uranus	19.2	23.8	49	59°	97.8°	N	17.3h	27
Neptune	30.0	22.2	27	47°	28.3°	R	16h	26

AU = Astronomical Unit = 1.5×10^8 km, ME = Earth's dipole moment = 8.05×10^{22} Amp m^2, N = normal polarity, R = reverse polarity, d =day, h = hour.

The magnetic fields of most planets are caused by internal dynamo mechanisms in the outer core of planets. These dynamos are driven by the thermal gradient between the inner core and the crust. Similarly the magnetic field on the sun is generated in a turbulent convection zone below the photosphere. However, the turbulent convection in the sun generates a much more turbulent magnetic field such that the field in solar prominences is much stronger than the dipole component.

In a plasma the magnetic field is usually much stronger than the electric fields. The reason for this is the effective shielding of electric fields by the collective behavior of electric charges. Vice versa the magnetic field generated by electric currents is not shielded and therefore a long range force field. It is instructive to examine the properties of electromagnetic field under Lorentz transformations.

$$\mathbf{E}' = \gamma_L \left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) - \left(\gamma_L - 1\right)\left(\mathbf{E}\cdot\mathbf{e}_v\right)\mathbf{e}_v \quad (3.1)$$

$$\mathbf{B}' = \gamma_L \left(\mathbf{B} - \frac{\mathbf{v}}{c^2}\times\mathbf{E}\right) - \left(\gamma_L - 1\right)\left(\mathbf{B}\cdot\mathbf{e}_v\right)\mathbf{e}_v \quad (3.2)$$

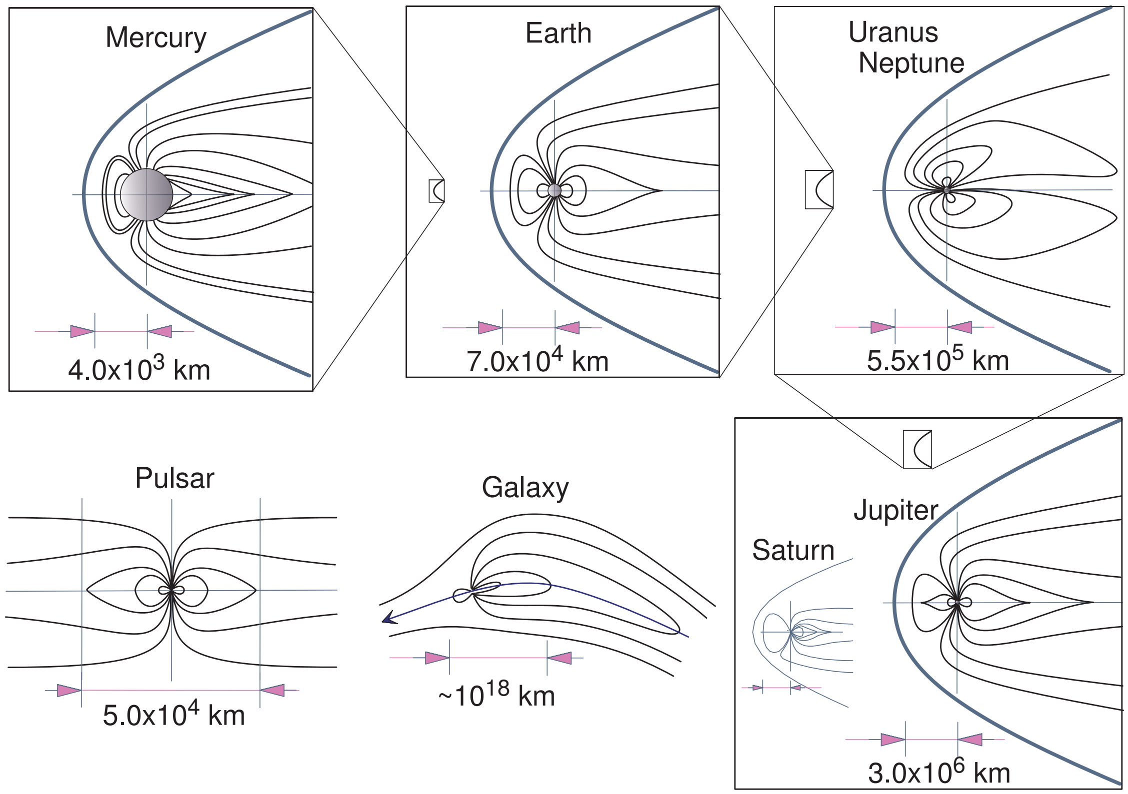

Figure 3.1: Comparison of different planetary magnetospheres.

with the Lorentz factor $\gamma_L = (1 - v^2/c^2)^{-1/2}$. In a plasma the electric field is often caused by convection and such it is of order O($|\mathbf{v} \times \mathbf{B}|$). Such an electric field magnitude yields a modification of order $\gamma_L v^2/c^2 \ll 1$ in the magnetic field. The Lorentz invariants of the electromagnetic field are

$$B^2 - E^2/c^2 = const \quad (3.3)$$

$$\mathbf{E} \cdot \mathbf{B} = const \quad (3.4)$$

The invariants demonstrate

- If the magnetic (electric) energy density is larger in one reference frame then it is larger in all reference frames.
- If there is an electric field component parallel to the magnetic field it exists in all reference frame.
- If the magnetic field is perpendicular to the electric field in one frame it is perpendicular in all reference frames.

3.1 Magnetic Fields

3.1.1 Representation

Maxwell's equations

$$\nabla \cdot \mathbf{B} = 0 \tag{3.5}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \tag{3.6}$$

Magnetostatics, Biot-Savard's law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3 r' \tag{3.7}$$

Vector potential: $\nabla \cdot \mathbf{B} = 0$ =>

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Gauge invariance: $\mathbf{A}$ -> $\mathbf{A}' = \mathbf{A} + \nabla \Xi$

Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

=> $\Delta \mathbf{A} = -\mu_0 \mathbf{j}$

Representation in vacuum fields $\mathbf{j} = 0$ => $\nabla \times \mathbf{B} = 0$ => $\mathbf{B} = -\nabla \Psi$

Euler potentials α and β: $\mathbf{A} = \alpha \nabla \beta + \nabla \Xi$ =>

$$\begin{aligned} \mathbf{B} = \nabla \times \mathbf{A} &= \nabla \times [\alpha \nabla \beta + \nabla \Xi] \\ &= \nabla \alpha \times \nabla \beta \end{aligned} \tag{3.8}$$

Note that $\mathbf{A} \cdot \mathbf{B} = 0$ is not generally satisfied but it is always possible to find a gauge such that $\mathbf{B}$ is perpendicular to $\mathbf{A}$.

Using Euler potentials the magnetic field is perpendicular to $\nabla \alpha$ and $\nabla \beta$ or - in other words - magnetic field lines are the lines where isosurfaces of α and β intersect.

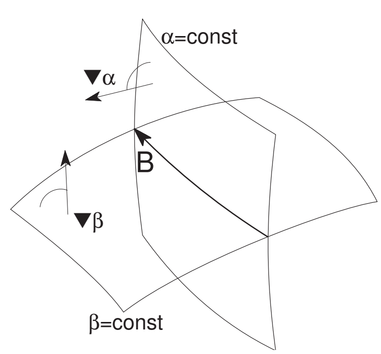

Figure 3.2: Sketch of the field interpretation of Euler potentials.

Significance of the magnetic field and field lines:

- Particles move easily along the magnetic field but are limited in the perpendicular direction. => Field lines can help to understand how particles access a certain region of space.
- The magnetic field is equivalent to the distribution of electric currents. Field lines and the convection (transport of magnetic flux) of field lines provides inside into configurational changes:
  - changes in the magnetic topology,
  - distribution of magnetic energy,
  - mixing of plasma
  - connection of different region of the magnetosphere.
- Field lines are an idealization invented to illustrate effects associated with the magnetic field. Do field lines exist?
  - Frozen-in condition required for identity.
  - Properties of magnetic flux tubes (finite width field lines).
- Mathematical representation $d\mathbf{l} \times \mathbf{B} = 0$ with $d\mathbf{l}$ line element. => $dx_i/dx_j = B_i/B_j$
- Implications of $\nabla \cdot \mathbf{B} = 0$
  - Ends of field lines?
  - Measure of the magnetic field (energy or magnetic flux), how much magnetic flux exists in the universe?(-> dynamo problem). Mathematical or theoretical concept must reflect physical reality and be of value for our physical interpretation.
- Existence of Euler potentials is questionable for tokamak configurations.

3.1.2 Dipole Magnetic Field

Vacuum field: $\mathbf{B} = -\nabla\Psi$

Earth magnetic dipole in spherical coordinates (r, θ, φ):

$$\Psi = -\frac{\mu_0 M_E}{4\pi}\frac{\cos\theta}{r^2} \tag{3.9}$$

with the latitude $\lambda = \pi/2 - \theta$ such that $\partial/\partial\theta = -\partial/\partial\lambda$ and the unit vector $\mathbf{e}_\theta = -\mathbf{e}_\lambda$. Noting that

$$\nabla\Psi = \frac{\partial\Psi}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial\Psi}{\partial\theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\varphi}\mathbf{e}_\varphi$$

the dipole magnetic field components become

$$B_r = -\frac{\partial\Psi}{\partial r} = -\frac{\mu_0 M_E}{2\pi}\frac{\sin\lambda}{r^3} \tag{3.10}$$

$$B_\lambda = -\frac{1}{r}\frac{\partial\Psi}{\partial\lambda} = \frac{\mu_0 M_E}{4\pi}\frac{\cos\lambda}{r^3} \tag{3.11}$$

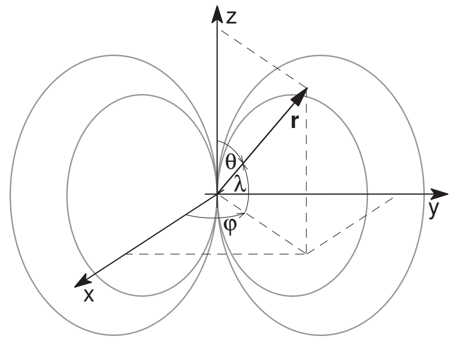

Figure 3.3: Dipole geometry.

Magnetic field magnitude:

$$B = \frac{\mu_0 M_E}{4\pi r^3}\left(1 + 3\sin^2\lambda\right)^{1/2}$$

At the equator on the Earth's surface the expression yields the magnetic field $B_E = \mu_0 M_E / 4\pi R_E^3$ which yields $B_E = 3.11 \cdot 10^{-5}$ T such that

$$B = B_E \frac{R_E^3}{r^3}\left(1 + 3\sin^2\lambda\right)^{1/2} \tag{3.12}$$

For diagnostic such as the study of particle orbits it is helpful to parameterize the dipole field lines. A line element $d\mathbf{s}$ of a field line satisfies $d\mathbf{s} \times \mathbf{B} = 0$ which leads to

$$\frac{dr}{B_r} = \frac{r d\lambda}{B_\lambda}$$

$$or \qquad \frac{dr}{r} = -2\frac{\sin\lambda}{\cos\lambda}d\lambda$$

The integral yields $\ln(r/r_0) = 2\ln(\cos\lambda/\cos\lambda_0)$. Starting the integration from the foot point in the equatorial plane gives $r = r_{eq}\cos^2\lambda$. It is customary to measure units in the magnetosphere in Earth radii $r_{eq} = r|_{\lambda=0} \equiv LR_E$ which yields

$$\tilde{r} = L\cos^2\lambda \tag{3.13}$$

where $\tilde{r}$ is measured in Earth radii and L is the so-called McIlwain L parameter. $\tilde{r} = 1$ gives the latitude of the foot point of a field line on the Earth's surface. => $\lambda_E = \arccos\sqrt{1/L}$.

Finally it is often helpful to know the variation of the field as a function of latitude and L value.

$$B(\lambda, L) = \frac{B_E}{L^3}\frac{\sqrt{1 + 3\sin^2\lambda}}{\cos^6\lambda} \tag{3.14}$$

3.1.3 Field Line Representation by the Vector potential

Cartesian geometry: In two dimensions the vector potential can be helpful to illustrate magnetic field lines. Assuming $\partial/\partial z = 0$ the magnetic field in the x, y plane can be represented by the z component of the vector potential

$$\mathbf{B}(x, y) = \nabla \times \mathbf{A} = \nabla \times A_z(x, y)\mathbf{e}_z + B_z(x, y)\mathbf{e}_z$$

We only need two independent variable because of $\nabla \cdot \mathbf{B} = 0$. Note also that this form of $\mathbf{B}$ always satisfies $\nabla \cdot \mathbf{B} = 0$. Denoting the field in the x, y plane as

$$\mathbf{B}_\perp = \nabla \times A_z(x, y)\mathbf{e}_z = \nabla A_z(x, y) \times \mathbf{e}_z \tag{3.15}$$

it follows that $\mathbf{B}_\perp$ is perpendicular to ∇A_z and $\mathbf{e}_z$.

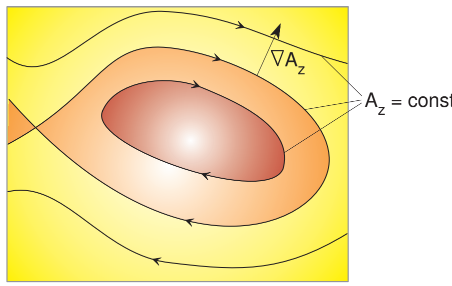

Figure 3.4: Representation of field lines by the the vector potential

Therefore lines of constant A_z (contour lines of A_z) are magnetic field lines projected into the x, y plane. The difference of the vector potential between two field lines is a direct measure of the magnetic flux bound by these field lines. The vector potential can be obtained by integrating

$B_x = \partial A_z/\partial y$ and $B_y = -\partial A_z/\partial x$. Finally it should be mentioned that this representation of field lines is a special case of Euler potentials with $\alpha = A_z$ and $\beta = z$.

Spherical coordinates: Again we assume two-dimensionality with $\partial/\partial\varphi = 0$. In this case the magnetic field is expressed as

$$\mathbf{B}(r, \theta) = \nabla \times A_\varphi(r, \theta)\mathbf{e}_\varphi + B_\varphi(r, \theta)\mathbf{e}_\varphi \tag{3.16}$$

Explicitly the magnetic field components are

$$B_r = \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} (r \sin\theta\, A_\varphi) \tag{3.17}$$

$$B_\theta = -\frac{1}{r \sin\theta} \frac{\partial}{\partial r} (r \sin\theta\, A_\varphi) \tag{3.18}$$

Remembering that the gradient in the r, θ plane is defined as

$$\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial\theta} e_\theta$$

one can choose $f = r \sin\theta\, A_\varphi$ such that $\mathbf{B}\cdot\nabla f = 0$. It follows that magnetic field lines are determined by

$$r \sin\theta\, A_\varphi = const \tag{3.19}$$

Applications to the dipole field: With

$$B_r = -\frac{\mu_0 M_E}{2\pi}\frac{\cos\theta}{r^3}$$
$$B_\theta = -\frac{\mu_0 M_E}{4\pi}\frac{\sin\theta}{r^3}$$

Using the magnetic field and integrating the equations (3.17) and (3.18) the vector potential A_ϕ becomes

$$A_\phi = -\kappa\frac{\sin\theta}{r^2} \tag{3.20}$$

with $\kappa = \mu_0 M_E/4\pi = B_E R_E^3$. Note that $r\sin\theta\, A_\varphi = const$ produces the already known equation for field lines as a function of r and θ. Let us now superimpose a constant "IMF" (interplanetary magnetic field along the z direction. (This is only justified for vacuum fields, i.e., in the absence of any currents. A simple superposition of magnetic fields in a plasma does not provide a self-consistent equilibrium solution.) The constant IMF $\mathbf{B}_{IMF} = B_0\mathbf{e}_z$ must be expressed in spherical coordinates: $B_r = B_0\cos\theta$ and $B_\theta = -B_0\sin\theta$ which yields

$$A_{\phi IMF} = \frac{B_0}{2} r\sin\theta \tag{3.21}$$

The superposition of the IMF and the dipole field is given by the sum of the corresponding vector potentials. The field lines for this configuration are then determined by

$$r^2\sin^2\theta\left(\frac{B_0}{2} - \frac{\kappa}{r^3}\right) = const \tag{3.22}$$

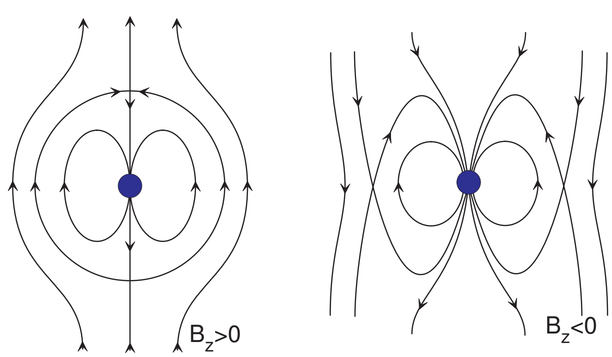

Figure 3.5: Superposition of a dipole field and constant IMF

The field geometry differs depending on whether B_0 is positive or negative. the two solutions are illustrated in Figure..

3.1.4 Local Magnetic Field Properties

One can categorize magnetic field structure and shape based on a local expansion of the magnetic field. For this purpose a constant field component B_0 along z is assumed such that the local expansion of the is

$$\mathbf{B}(x, y, z) = B_0\mathbf{e}_z + \mathbf{r}\cdot\nabla\mathbf{B} \tag{3.23}$$

or $B_j(x, y, z) = B_0\delta_{jz} + r_i\partial_i B_j$ where the derivative $\partial/\partial r_i$ is abbreviated by ∂_i and the summation convention is implied, i.e., when the same index appears twice it is implied to take the sum over this index such that $r_i\partial_i$ is the operator $x\partial/\partial x + y\partial/\partial y + z\partial/\partial z$. We can now systematically study the various terms of this expansion and their implications.

a) Diagonal terms:

Assuming that only $\partial_x B_x$, $\partial_y B_y$, and $\partial_z B_z$ are nonzero the x and y field components become

$$B_x = x\partial_x B_x|_0, \quad B_y = y\partial_y B_y|_0, \quad B_z = B_0 \tag{3.24}$$

where $|_0$ implies expansion at the origin. Thus the field equations become

$$\frac{dx}{dz} = \frac{B_x}{B_z} = x\frac{\partial_x B_x|_0}{B_0}$$

and similar for dy/dz. Defining $\alpha_x = \frac{\partial_x B_x|_0}{B_0}$ the field equations are

$$x = x_0 \exp \alpha_x z \quad and \quad y = y_0 \exp \alpha_y z \tag{3.25}$$

Depending of the sign of α_x and α_y the field lines are converging or diverging. Note that for the diagonal elements $\nabla \cdot \mathbf{B} = 0$ must be satisfied.

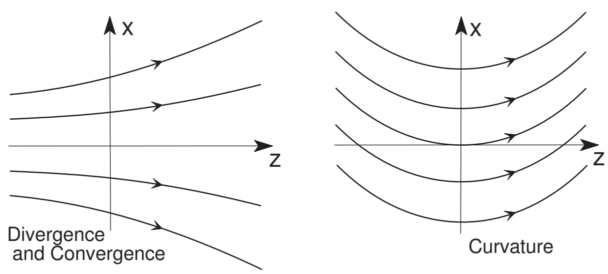

Figure 3.6: Illustration of divergence and curvature of magnetic fields.

b) Terms $\partial_z B_x$, $\partial_z B_y$:

In this case the field equations are determined by

$$\frac{dx}{dz} = z\frac{\partial_z B_x|_0}{B_0} \quad and \quad \frac{dy}{dz} = z\frac{\partial_z B_y|_0}{B_0} \tag{3.26}$$

With the definition $\beta_x = \partial_z B_x|_0/B_0$ and $\beta_y = \partial_z B_y|_0/B_0$ the field equations are

$$x = x_0 + \frac{1}{2}\beta_x z^2 \quad and \quad y = y_0 + \frac{1}{2}\beta_y z^2 \tag{3.27}$$

These equations imply locally parabolic field lines and therefore the corresponding terms imply magnetic field curvature.

c) Terms $\partial_x B_z$, $\partial_y B_z$:

In this case the derivative of B_x and B_y are 0 and thus these components are constant. Only B_z changes in magnitude in the x and y directions. Therefore these terms imply a gradient in the magnetic field in the form $B_z = B_0 + \delta_x x$ with $\delta_x = \partial_x B_z/B_0$ and similar for the y derivative.

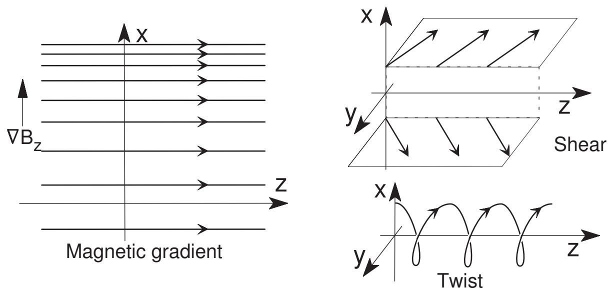

Figure 3.7: Illustrations of magnetic gradient, shear, and twist.

d) Terms $\partial_y B_x$, $\partial_x B_y$:

Let us first assume that $\partial_y B_x = 0$ implying $B_x = 0$. The field line equation for B_y becomes

$$\frac{dy}{dz} = x\frac{\partial_x B_y|_0}{B_0} \tag{3.28}$$

With $\gamma = \partial_x B_y|_0/B_0$ the solution is

$$x = const \quad y = \gamma xz + y_0 \tag{3.29}$$

In this case the field lines have no x component. For each value of x they represent straight lines parallel to the y, z plane. However, the slope in the y, z plane changes with the x. Thus the field lines are sheared. If both terms $\partial_y B_x$**,** $\partial_x B_y$ are nonzero field lines can become helical and in general a current is associate with this (and the sheared) field geometry.

In Summary:

A magnetic field can locally be expanded $\mathbf{B}(x, y, z) = B_0\mathbf{e}_z + \mathbf{r}\cdot\nabla\mathbf{B}$ with

$$\nabla\mathbf{B} \;=\; B_0\begin{pmatrix} \alpha_x & \gamma & \delta_x \\ \gamma' & \alpha_y & \delta_y \\ \beta_x & \beta_y & \alpha_z \end{pmatrix} \tag{3.30}$$

where the terms α_i imply divergence and convergence, β_i curvature, γ_i shear and twist, and δ_i gradients of the magnetic field. Note that the elements of $\nabla\mathbf{B}$ are subject to the constraint of $\nabla\cdot\mathbf{B} = 0$ and they determine the current density in the particular location.

Exercise: Determine the condition for α_i to satisfy $\nabla\cdot\mathbf{B} = 0$. Assume a vacuum field. Determine the constraints to the elements of $\nabla\mathbf{B}$. Show that in a vacuum magnetic field a magnetic gradient always implies the presence of curvature in a magnetic field. Discuss Shear and twist in a vacuum magnetic field.

3.1.5 Magnetic Fields and Electrical Current

a) Tail-like field geometry: As a rather coarse approximation of the magnetotail current let us consider the following magnetic field (in a suitable normalization).

$$\mathbf{B} = -z\mathbf{e}_x + d\mathbf{e}_z \tag{3.31}$$

The field line equation $dx/dz = B_x/B_z = -z/d$. The solution to these equations are the parabolas

$$x = -\frac{1}{2d}z^2 + const \tag{3.32}$$

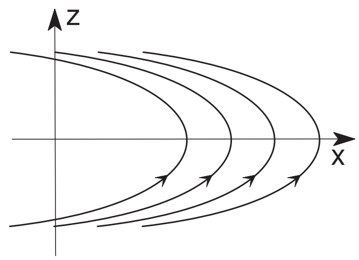

Figure 3.8: Sketch of the tail-like magnetic field.

which resemble a tail-like geometry. The current density for this configuration is

$$\begin{aligned} j_x,\, j_z &= 0 \\ \mu_0 j_y &= \partial_z B_x - \partial_x B_z = -1 \end{aligned}$$

Note that the same result for the field lines can be obtained using the vector potential component A_y from $\mathbf{B} = \nabla A_y \times \mathbf{e}_y$ such that

$$\begin{aligned} B_x = -z = -\partial_z A_y \quad &=> \quad A_y = \frac{1}{2}z^2 + f(x) \\ B_z = d = \partial_x A_y \quad &=> \quad A_y = dx + g(z) \end{aligned}$$

Therefore the complete solution for the vector potential is

$$A_y = \frac{1}{2}z^2 + dx + c_0 \tag{3.33}$$

Since field lines are given by $A_y = const$ we arrive at $x = -\frac{1}{2d}z^2 + const$ as before.

b) Magnetic O and X lines:

Let us now consider the case

$$\mathbf{B} = y\mathbf{e}_x \pm \beta^2 x\mathbf{e}_y \tag{3.34}$$

The field equation for this case is $dx/dy = B_x/B_y = \pm y/\beta^2 x$ yielding the solution

$$\pm\beta^2\left(x^2 - x_0^2\right) = y^2 - y_0^2 \tag{3.35}$$

The corresponding field lines are sketched in Figure 3.9. For the solution with the $+$ sign the resulting field lines produce the X type configuration on the left in the figure. For the - sign in equation (3.35) the result are elliptical shaped field lines as shown on the right in Figure 3.9. In the figure, the X line configuration has 4 field lines connected to the origin (the so-called X point or X line -considering the third dimension). The X line and also the O line in the other configuration are also called neutral points

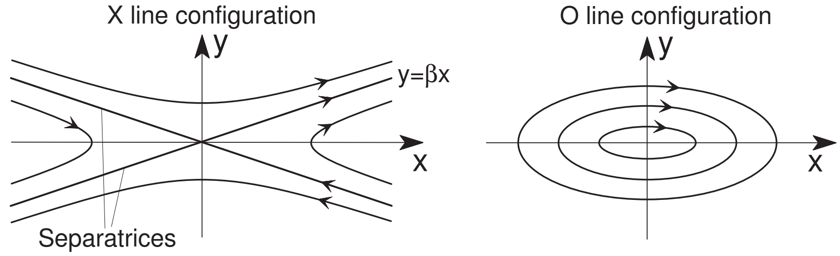

Figure 3.9: Illustration of X type and O type neutral point configurations.

because the magnetic (in this plane) is 0. The field lines connected to the X are called separatrices. The current density for these configurations is

$$\mu_0 j_z = \pm\beta^2 - 1 \tag{3.36}$$

Note that the current density for the + configuration becomes 0 for $\beta = \pm 1$ which represents the case of a vacuum field. The + configuration represents an X line similar to the case magnetic reconnection which will be discussed in a later chapter. Reconnection, however, requires a nonzero current to proceed such the the case with slopes of ± 1 for the separatrices cannot represent magnetic reconnection.

3.2 Electric Fields

Maxwell equations:

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho_c \tag{3.37}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{3.38}$$

Potential representation:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi \tag{3.39}$$

However, quasi-neutrality implies $\rho_c \approx 0$ such and (3.37) is substituted by Ohm's law. Sometimes it is argued that Ohm's law can be derived from the Lorentz transformation (3.1) for non-relativistic plasmas ($v \ll c$ and $\gamma_L = 1$) from a frame in which the electric field is $\mathbf{E}' = 0$ such that

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

However, this can be done only at locations $B^2 - E^2/c^2 > 0$ and if $\mathbf{E} \cdot \mathbf{B} = 0$ because the transformation of the electric field is subject to the Lorentz invariants. Even if we find a transformation velocity to a frame where the electric field is 0 locally, this does not need to be a rest frame of the plasma because other terms in ohms law such as the inertial and pressure gradient terms can generate an electric field. Furthermore the electric and magnetic fields typically are not constant such that there is no frame of reference in which the electric field is 0 everywhere. Therefore, Ohm's law is in fact determined by the fluid momentum equations (mostly the electron equation) as derived in equation (2.31)!

Stationary states: A steady state is characterized by $\partial/\partial t = 0$

Particularly

$$\frac{\partial \mathbf{B}}{\partial t} = 0 \quad => \quad \nabla \times \mathbf{E} = 0$$

or

$$\mathbf{E} = -\nabla \phi \tag{3.40}$$

A frequently used special case is $\mathbf{E} = const$.

Examples:

i) Magnetospheric convection

Southward IMF: $\mathbf{v}_{sw} = -v_0 \mathbf{e}_x$, $\mathbf{B}_{sw} = -B_0 \mathbf{e}_z$ **=>** $\mathbf{E} = -\mathbf{v} \times \mathbf{B} = v_0 B_0 \mathbf{e}_y$

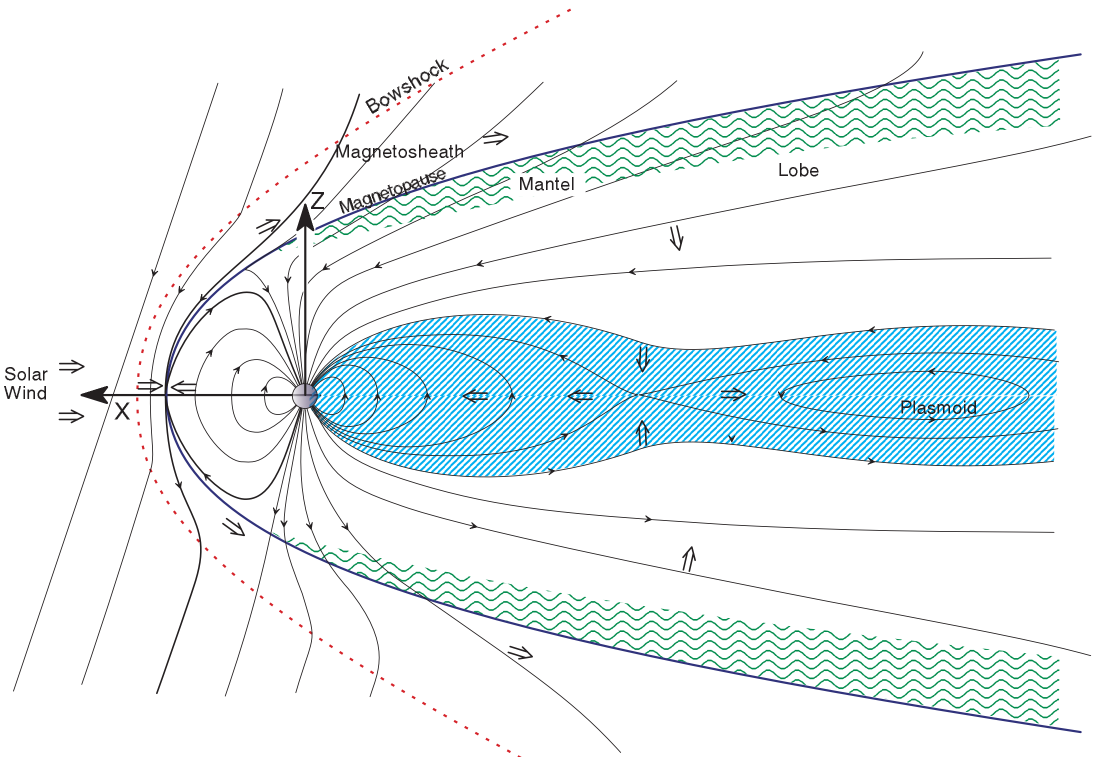

Figure 3.10: Convection in the magnetosphere for southward IMF.

The resulting convection as is illustrated in Figure 3.10 yields

- Plasma flow toward the plasma sheet in the tail.
- Earthward jetting in the plasma sheet.
- Plasmoid ejection (but tailward end of the plasmoid should move earthward if the model were accurate).
- Flow toward the dayside magnetopause.

ii) Convection in the inner magnetosphere due to co-rotation of the magnetic field.

$\mathbf{v} = \omega_E \mathbf{e}_\omega \times \mathbf{r}$ such that $\mathbf{E} = -\omega_E \left(\mathbf{e}_\omega \times \mathbf{r}\right) \times \mathbf{B}_{dip}$.

iii) Stationary magnetic reconnection models.

Remarks on the assumption of a steady state, constant electric field or electric field mapping:

The assumption of a steady state configuration can yield helpful insight. However, it should be remembered that any such assumption can only be valid in a limited region of space and for a limited duration. More precisely, considering a process or configuration that changes on a time scale τ which is long compared to the transit time of typical wave through the configuration we call this process slow.

For instance, the flow of a gas or liquid through a hose usually has a velocity much slower than the speed of sound, such that it is justified to assume a constant flow rate through the hose. In such a case it can be justified to consider the steady state description of this flow. However, not all slow processes are well described by a steady state. For instance, inflating a balloon takes usually much longer than the travel time of a sound wave through the balloon. However, the balloon increases in size, pressure increases, and particularly the surface tension increases. While the airflow may be at a steady rate the configurational changes are significant and can cause the balloon to burst when tensions are to high.

We also need to consider the typical temporal and spatial scales which are used to describe a processes. An example are the properties of a shock wave. On sufficiently large scales we use a steady state fluid description to determine jump conditions, i.e., the properties of the gas or plasma downstream of the shock such as compression, velocity, or magnetic field. This is perfectly justified on these scales, however, we know that a shock is a highly dynamic structure with large fluctuations on small 'kinetic' scales. A similar example is the modeling of magnetic reconnection.

It is often assumed or argued that the SW or better the dayside reconnection electric field penetrates into the magnetosphere to set up convection. However, large scale reconnection on the nightside typically occurs about 1/2 to 1 hours after reconnection had started on the dayside. During this time, the amount of open magnetic flux changes strongly and the outer magnetosphere undergoes a major reconfiguration. Clearly the evolution cannot be captured by a steady state description which requires $\partial \mathbf{B}/\partial t = 0$. Therefore, convection is truly a result of the forces and force balance in a system. For instance, the tailward end of the plasmoid which is formed during reconnection in the tail should move Earthward if the electric field were constant. This is not only counter intuitive but not consistent with observations or quantitative models.

This discussion has implications for ionospheric convection. In a steady state the electric field can be derived from a potential. A typical (and often well justified) assumption is that the electric field component along the magnetic field is 0 or small, which implies the magnetic field lines are equipotential lines. Furthermore, the approximation of the ionospheric convection field as a potential field is well justified in the polar ionosphere with a strongly vertical magnetic field because the magnetic field magnitude changes very little such that $\partial \mathbf{B}/\partial t = 0$. Also the wave speed (fast mode) is very large compared to convection velocities such that horizontal convection is largely incompressible consistent with the assumption of a potential electric field. However, one ought to be careful to map this field out into the magnetosphere because this would require that convection in the outer magnetosphere is also in a steady state with a consistent potential electric field.

Exercise: Assume a SW velocity of 400 km/s, a southward IMF component of 5 nT. Compute the convection velocity in the northern tail lobe if the magnetic field is entirely in the positive x

direction and has a value of 20 nT. Compute the convection speed in the center of the plasma sheet assuming a field of 1 nT in the positive z direction (these are typical values for the mid-tail).

Exercise: Frequently physical effects are attributed to the Poynting flux and in particular to a convergent Poynting flux. Assume ideal Ohm's law and compute the Poynting flux for $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$. Interpret your result.