3. Debye Screening

(a) Demonstrate that the plasma definition $\langle e\phi \rangle \ll \left\langle \frac{m}{2} v^2 \right\rangle = k_B T$ implies quasi-neutrality, i.e., $\Lambda = n\lambda_D^3 \gg 1$.

(b) The plasma parameter has the basic dependence $\Lambda \propto n_0^{-1/2} T^{3/2}$. While the dependence on temperature is intuitively clear, the density dependence appears odd because lower densities seems to imply fewer particles and less shielding. Why is intuition wrong and why does the plasma parameter improve (increase) with decreasing density?

4. Plasma properties

a) Assume a plasma density of 1 cm^{-3}, temperature equivalent to 1 keV, and a magnetic field of 20 nT which are typical for the near Earth magnetotail. Determine Debye radius and the plasma parameter Λ.

b) Compute the the electron plasma frequency, the collision frequency for electrons, the thermal velocity, and the mean free path of an electron based on these numbers. Can one neglect collisions for these electrons for the length scales of the magnetosphere of 100 R_E? (1 R_E=6400 km)

5. Plasma energy

(a) For the plasma in problem 4, determine the temperature in degrees Kelvin, and the thermal and magnetic energy density. How do these compare?

(b) Express the energy densities in kW hours/m^3 and kW hours /R_E^3. For the sake of simplicity assume that the plasma sheet is represented by a cylinder with 15 R_E radius and 100 R_E length. How long could a power plant with an output of 1000 MW operate on the thermal energy stored in the plasma?

6. Collisionless Boltzmann equation

(a) Show that any distribution function $f(\mathbf{x}, \mathbf{v}, t) = F(H, P_y)$ with $H = m\mathbf{v}^2/2 + q\phi(\mathbf{r})$ and $P_y = mv_y + qA_y$ solves the steady state ($\partial/\partial t = 0$) collisionless Boltzmann equation for $\partial/\partial y = 0$.

(b) Consider a distribution function of the form $F(H) = c_0 \exp(-H/k_B T)$. Show that the plasma density is given by $n(\mathbf{r}) = n_0 \exp(-q\phi/k_B T)$ and express c_0 in terms of n_0 and $k_B T$.

Please turn in the solutions to the homework on Friday, 02/07/2014