

7. Plasma definition

Can a plasma be maintained at temperatures of $T_e = 100$ K (Hint: Calculate the density limit using the plasma parameter and explain your result).

8. Moments of a distribution function

A Maxwellian velocity distribution function is given by

$$f(\mathbf{v}) = n \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp \left(-\frac{m}{2k_B T} [v_x^2 + v_y^2 + (v_z - v_{z0})^2] \right)$$

Compute the average (bulk) velocity of particles described by the distribution function. The kinetic energy can be split into a thermal portion and a part which is caused by the bulk motion of particles. Compute the thermal and the bulk kinetic energy? Determine the value of the bulk velocity for which the bulk kinetic energy equals the thermal energy.

9. MHD equations

Assume a scalar pressure, $\mathbf{L} = 0$, $Q^E = 0$, and $Q^p = 0$. Consider a function $h(\rho p) = \rho^a p^b$ and determine a and b such that the resulting equation for h assumes a total derivative, i.e., $\partial h / \partial t + \mathbf{u} \cdot \nabla h = 0$. For $\gamma = 5/3$ this becomes the equation for an entropy function because entropy is conserved for adiabatic changes. (Hint: Use the continuity and the pressure equation to eliminate the time derivatives of ρ and p).

10. Normalization

- Following the example presented in class, determine typical values for the electric field v_0 , E_0 , and the pressure p_0 from the MHD equations in terms of L_0 , ρ_0 , B_0 .
- Using the normalization procedure, derive the coefficients of the inertial term and of the Hall term in generalized Ohm's law. Show that these coefficients are $(c/\omega_{pe})^2 / L_0^2$ and $c / (\omega_{pi} L_0)$ respectively.
- What are the values of these coefficients for the plasma parameters from problem 4 and $L_0 = 1 R_E$?