14. Local expansion of B: Consider the matrix

$$\nabla \mathbf{B} \;=\; \begin{pmatrix} \alpha_x & \gamma & \delta_x \\ \gamma' & \alpha_y & \delta_y \\ \beta_x & \beta_y & \alpha_z \end{pmatrix} \tag{1}$$

with constant parameters α_i, β_i, δ_i, γ, and γ' for the expansion of the magnetic field at the origin.

(a) Assume the magnetic field at the origin $\mathbf{r} = 0$ has only a B_z component B_0. Determine the magnetic field in the vicinity of the origin to first order in x, y, and z.

(b) Derive the most general form of the matrix elements for a static vacuum magnetic field $\mathbf{j} = 0$ and satisfying $\nabla \cdot \mathbf{B} = 0$.

(c) Using the above result, demonstrate that the presence of nonzero curvature in a static vacuum field always implies the presence of a nonzero magnetic gradient and vice versa.

15. Current layer magnetic field: Consider a magnetic field given by $\mathbf{B} = y\mathbf{e}_x + \alpha \sin(kx)\,\mathbf{e}_y$ and assume $\alpha,\ k > 0$.

(a) Derive the equations for the field lines.

(b) Determine the vector potential and the two Euler potentials.

(c) Sketch and discuss the field lines (Hint: determine the location of X and O lines first).

(d) Determine the condition for α and k such that the the current density at X lines is 0.

16. Radius of curvature: A magnetic field is given by $B_z = \epsilon B_0$, $B_x = B_0 z/L$.

(a) Compute the radius of curvature as a function of z, ϵ and L and show that the radius of curvature is $r_c = \epsilon L$ at $z = 0$.

(b) Use $L = 1\,R_E$, and $\epsilon = 0.1$ to determine the centrifugal acceleration for 10 keV (parallel energy) electrons and protons at $z = 0$ in the magnetotail.

(c) Compute the resulting curvature drift velocity for these particles for $B_0 = 20\,nT$.

17. Loss cone: Calculate the size of the loss cone at the geomagnetic equator for particles on a dipole magnetic field line whose equatorial crossing distance is $5R_E$ ($1\ R_E = 6400km$). Assume the particles are mirrored in the ionosphere at an altitude of 100 km from the surface. How large is the difference in the loss cone if particles are lost at 1000 km altitude?

Please turn in the solutions to the homework on Monday, 02/28/14