21. Energy conservation

Derive the conservative form of the energy equation in MHD.

22. Sound waves

a) Reduce the MHD equations to a non-magnetic fluid. Determine the equilibrium conditions.

b) Consider the pressure and continuity equations. Explain why a choice of $\gamma = \infty$ corresponds to incompressibility and why $\gamma = 1$ corresponds to isothermal changes (γ is the ratio of specific heats)?

c) Assume a homogeneous system (constant pressure and density, and velocity $\mathbf{u} = 0$) and derive the dispersion relation for sound waves by linearizing the equations and assuming waves of the form $f(x,t) = f_0 \exp\{i(kx - \omega t)\}$. The corresponding waves are sound waves. What is the wave speed?

23. Simulation of sound waves

The compressed file mhd1d_code on the class website contains a one-dimensional MHD simulation code, files to run the program, an idl program to visualize results, a readme file that explains how to run the code, and a few page of background on the MHD simulation. The one-dimensional MHD code contains various initial conditions. Make yourself familiar with the program by running it for the case of a sound wave (the preset initial condition 3 -> subroutine initc3). Compile and run the program in the distributed version. Examine the results using the provided idl program (Note the system is periodic such that the wave that exits to the right re-enters from the left). (1) Is it possible to get rid of the oscillations at later times by increasing one of the viscosity parameters in m1in (ivisrho, ivisv, ivisu)? (2) What happens when you change the wave velocity amplitude to 0.01 or the density to 4 (in the program)? (3) How large can you make the time step before you encounter an instability? Note that you may have to use a shorter run (smaller iend) and more frequent outputs (smaller iout) to see this. Report your results with selected plots that document your findings and attempt an interpretation of your findings.

Please turn in the solutions to the homework on Friday, 3/14/2014