30. Switch-off shock - Properties

Switch-off shocks are an important structure in simple models for magnetic reconnection.

(a) Compute the compression ratio for the limiting cases of the shock angles $\theta = 0$ and $\theta = \pi/2$.

(b) The dimensionless reconnection rate r is typically of $O(0.1)$ and is approximately $r = \cos\theta$ (i.e., θ is close to $\pi/2$). Expand the solution for the compression ratio around $\theta = \pi/2$ to include the lowest order correction for r and show that the compression ratio is approximately

$$X = 1 + \frac{1}{\gamma\beta + \gamma - 1}\left(1 - \frac{(\gamma\beta - 1)}{\gamma(\beta + 1)} r^2\right)$$

(c) Show that the amplification of the thermal pressure is given by

$$\frac{p_d}{p_u} = X\left[1 + \frac{\gamma - 1}{\gamma\beta}\left(1 - \frac{r^2}{X^2}\right)\right]$$

and compute the ratio of the entropy function $s = p/\rho^\gamma$.

(d) What are the compression, the pressure, and the entropy ratio in the limit of $\beta = 0$ and $\beta \gg 1$? Can you get large ratios for the entropy function in the case of high plasma β?

31. Switch-off shock - Simulation

Modify the the initial condition 1 to introduce a separate equilibrium density, pressure and magnetic field such that the initial configuration becomes $B_y = B_0 \tanh x$, $\rho = \rho_0 + B_0^2/\cosh^2 x$ and use $p = p_0 + B_0^2/\cosh^2 x$ and choose $B_0 = 1$, $\rho_0 = 1$, and $p_0 = 1$ as a reference. Run this case for with $B_x = 0$ and $B_x = 0.1$ (note, you have to run this probably for several 100 simulation times to see the evolution clearly, and you need to use `intu` $= 2$).

(a) The two step waves that propagate in the positive and negative x direction are slow switch-off shocks. Can you identify the plasma acceleration? Examine the properties and compare these with the results for the plasma compression and pressure ratio.

(b) Sketch the situation in the x, y plane. What transformation velocity is needed for a transformation into a frame in which the electric field is 0?

(c) What changes are needed to simulate low and a high β cases. Apply these changes and run cases with $\beta = 0.1$ and $\beta = 10$. How are the results different and Aare they consistent with the prediction for the low and high beta limits for the compression and pressure ratio in problem 30?

(d) Run the low beta case for `intu` $= 0$ and 2. How do the results differ? Can you explain why they are different?

32. Project:

Continue the work on your project. Provide another progress report for this week.

Please turn in the solutions to the homework on Friday, 4/18/14