33. Fast wave dispersion relation

(a) Consider a plasma with a uniform magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_y$, density ρ_0, and pressure p_0 at rest. Using the ideal MHD equations, derive the dispersion relation for a fast mode wave with it's wave vector $\mathbf{k}$ entirely along the x direction. What is the physical cause for this effect?

(b) Assume a magnetic field perturbation $\delta B_y = h(x,t)\, B_0$ with the wave shape function (or profile) $h(x,t) = h(x - x_0 + v_f t)$ and the fast mode speed $v_f = \sqrt{(\gamma p_0 + B_0^2/\mu_0)/\rho_0}$. Note that $h(s)$ can have any shape but it maximum should be smaller than 1 - otherwise a shock will form immediately. Using the MHD equations show that

$$\begin{aligned} u_x &= h(x,t)\, v_f \\ \rho &= (1 + h(x,t))\, \rho_0 \\ p &= (1 + \gamma h(x,t))\, p_0 \end{aligned}$$

34. Fast wave propagation + interaction with a plasma boundary

(a) Use the 1D MHD code with the initial condition 5. This condition uses a bell shaped waveform with $h = \widehat{h}/\cosh\left[(x - v_f t)/l_0\right]$ with the width l_0 and the amplitude $\widehat{h}$. Run this for $l_0 = 5$ and amplitude between 0.01, 0.1, and 1. Does the wave for small amplitudes satisfy the dispersion relation? How fast do you observe the steepening of the fast wave to a shock for the different perturbation? For the amplitude of $\widehat{h} = 1$ check the jump conditions for the fast perpendicular shock for the integration methods $\texttt{intu} = 0$ and 2.

(b) Now change the initial conditions to center the perturbation at $x = -20$ with $l_0 = 4$ and introduce a density boundary where the density decreases to 0.25 for $x > 0$ (this boundary can represent the magnetopause). Run the program for an amplitude of $\widehat{h} = 0.5$. What happens to the wave and the density boundary when the wave pulse interacts with this boundary? Repeat this for an amplitude of $\widehat{h} = 1$. Explain why the density boundary is displaced in this experiment. How would you compute this displacement?

Note: You may have to limit the time of the simulation to a few 10 Alfven time units; you may have to decrease the time step; and you need to check the boundary conditions (for the density jump they should not be periodic).

35. Project

Continue work on your project. Try to identify the major motivation, methodology, and the main results and provide a brief (~2 pages) summary of your findings. Formulate issues that you do not understand.

Please turn in the solutions to the homework on Friday, 4/25/2014