1. Distribution function

Consider a Bi-Maxwellian distribution function $f\left(v_\parallel, v_\perp\right) = c\exp\left[-\frac{m}{2}\left(\frac{v_\parallel^2}{k_B T_\parallel} + \frac{v_\perp^2}{k_B T_\perp}\right)\right]$ where $v_\parallel$ is along the magnetic field (assume the z direction) and $v_\perp$ is perpendicular to the field ($v_\perp^2 = v_x^2 + v_y^2$).

(a) Demonstrate that the normalization coefficient from $n = \int f d^3v$ is

$$c = n\left(\frac{m}{2\pi k_B}\right)^{3/2}\frac{1}{T_\parallel^{1/2}T_\perp}$$

(b) Compute the components of the pressure tensor.

(c) What is the ratio of $T_\parallel/T_\perp$ if the average gradient and the curvature drifts are equal in a vacuum magnetic field.

2. Magnetic field

Consider the magnetic field $B_x = B_0\tanh\left(z/L\right)$, $B_z = \epsilon B_0 x/L$, and $B_y = 0$ with $\epsilon > 0$.

(a) Compute the y component of the vector potential.

(b) Determine the equation for magnetic field lines.

(c) Discuss and sketch the field lines. What is the separatrix angle at the X line?

(d) Compute the current density associated with the magnetic field.

(e) Compute the acceleration as a function of x along the x axis for a plasma at rest (Use the MHD momentum equation and constant pressure). What is this acceleration for $B_0 = 20$ nT, $\epsilon = 0.1$, $L = 1\,R_E$, and density $n = 1$ cm^{-3} (protons and electrons) at $x = L$?

3. **Ohm's law and induction equation**

Assume $\mathbf{E} = -\mathbf{u}\times\mathbf{B}$, Faraday's law, and the field amplitude $B = |\mathbf{B}|$.

(a) What assumption for the flow or for the magnetic field is necessary to obtain

$$\frac{\partial B}{\partial t} + \nabla\cdot\left(\mathbf{u}B\right) = 0 \tag{1}$$

(b) Using the continuity equation show that (1) implies

$$\frac{d\left(\rho/B\right)}{dt} = 0$$

(c) Explain the physical meaning of these two equations.

4. Single particle drifts

List **(a)** 3 single particle drifts, **(b)** the basic physical mechanisms which cause the individual drifts, **(c)** the main assumptions which enter into the drift approximations, **(d)** an application for each particular drift where it plays an important role.

Help:

$$\int_0^\infty \exp\left(-a^2x^2\right)dx = \frac{\sqrt{\pi}}{2a}$$