Chapter 1

Basic Elements of the Physics of Charged Particles

1.1 Preliminaries

- Contact:
  - Antonius Otto
  - Geophysical Institute, 708E?
  - Phone 474 6169
  - Email: ao@how.gi.alaska.edu
  - Web site: www.what.gi.alaska.edu/ao/plasma
  - Office hours: Anytime
- Student names and background
- Encourage participation, criticism, and suggestions
- Scope of the course and contents (see handout or web page)
  - Basic elements of the physics of charged particles
  - Single Particle Motion
  - Fluid Equations
  - Magnetohydrodynamics
  - MHD Equilibria and Stability
  - Plasma Kinetic Equations and Collisions
  - Electrostatic Plasma Waves
  - Electromagnetic Plasma Waves

- Vlasov Equations and Equilibria
- Kinetic Instability
- Nonlinear Electrostatic Waves
- Weak Turbulence Theory

- Conduct (see handout or web page)
  - Textbooks, some lecture notes on the web, but emphasis on lecture notes
  - Homework: analytical
  - Grading
  - Midterm test and final exam
- Questions

1.2 Basic Properties and Definitions

1.2.1 Plasma definition:

- A plasma is a (partially) ionized gas in which the potential energy of a particle due to its nearest neighbor force is much smaller than its kinetic energy.

Applications and examples of plasma environments (All environments of partially or fully ionized gases)

- Laboratory applications
  - Fusion devices (inertial fusion or laser plasma , contained plasma in tokamaks ..)
  - Sputtering
  - Gas discharges
- Natural environment
  - Electric phenomena in atmosphere
  - Ionosphere
  - Planetary magnetospheres
  - Space (solar wind, there is no vacuum)
  - Stellar atmosphere
  - Stellar interior
  - Interstellar and intergalactic clouds

- Metal
- Dusty plasmas

The last two examples actually fit only partially the plasma definition. Far the case of metals individual atoms can certainly not move freely but the electrons usually can. For dusty plasmas (consisting of charge carrying dust grains which occur at any locations (atmosphere house dust, planetary rings etc.) the plasma definition is often only marginally satisfied (and sometimes not satisfied at all).

More than 99% of all visible matter in the universe is in the plasma state. Plasma densities range from 10^{-3} cm^{-3} (Jovian magnetosphere) to 10^{27} cm^{-3} (stellar interior). Plasma temperatures range from few 10^2 K (upper atmosphere) to 10^8 K in thermonuclear plasma

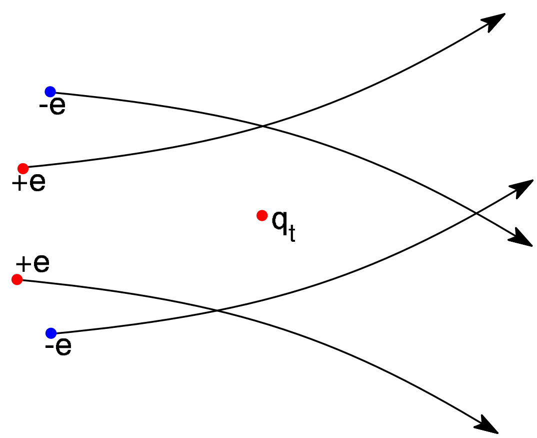
1.2.2 Debye Shielding and Plasma Parameter

A plasma is a gas of charged particles, which consists of free positive and/or negative charge carriers.

Coulomb potential of a test charge q_t at $\mathbf{r} = 0$:

$$\phi_D = q_t/(4\pi\epsilon_0 r). \tag{1.1}$$

In a plasma the test charge (of infinite mass) modifies the distribution of the particles in its vicinity.



Distribution function from equilibrium statistical mechanics

$$f_s(\mathbf{r}, \mathbf{v}) = const\ \exp(-H_s/k_B T_s) \tag{1.2}$$

with Hamiltonian $H_s = m_s v^2/2 + q_s \Phi$ yields the density distributions

$$\begin{aligned} n_i &= n_0 \exp(-e\Phi/k_B T_e) \\ n_e &= n_0 \exp(e\Phi/k_B T_i) \end{aligned}$$

Poisson's equation:

$$-\nabla^2\Phi = \frac{1}{\epsilon_0}\rho_c(\mathbf{r},t) = \frac{1}{\epsilon_0}\left[q_t\delta(\mathbf{r}) + e(n_i - n_e)\right] \tag{1.3}$$

Assume $e\Phi \ll \{k_BT_e, k_BT_i\}$ such that $\exp(e\Phi/k_BT_e) \approx 1 + e\Phi/k_BT_e$

=>

$$\nabla^2\Phi = -\frac{1}{\epsilon_0}q_t\delta(\mathbf{r}) + \frac{1}{\epsilon_0}e^2n_0\left(\frac{1}{T_e} + \frac{1}{T_i}\right)\Phi$$

Define the **Debye length** as

$$\lambda_D^{-2} = \frac{n_0e^2}{\epsilon_0k_B}\left(\frac{1}{T_e} + \frac{1}{T_i}\right) \tag{1.4}$$

such that

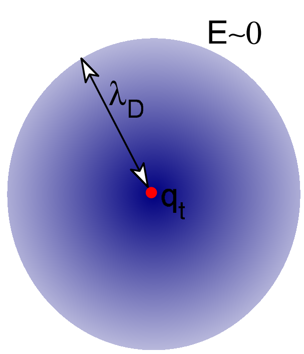
$$\nabla^2\Phi = -\frac{1}{\epsilon_0}q_t\delta(\mathbf{r}) + \frac{1}{\lambda_D^2}\Phi(\mathbf{r})$$

At large distances $\Phi \propto \exp(-r/\lambda_D)$

Full solution:

$$\phi_D = \frac{q_t}{4\pi\epsilon_0r}\exp\left(-\frac{r}{\lambda_D}\right) \tag{1.5}$$

This potential is sometimes called the Yukawa potential. In comparison to the coulomb potential the Yukawa potential converges much faster to 0 for length scales larger than the Debye length due to the exponential factor. Thus the electric field tends to 0 much faster or in other words the electric field from the test charge is effectively shielded at distances larger than the Debye length.



Defining the ion/electron Debye length $\lambda_{Di,e} = \left(\frac{\epsilon_0k_BT_{i,e}}{n_0e^2}\right)^{1/2}$ we have $\lambda_D^{-2} = \lambda_{Di}^{-2} + \lambda_{De}^{-2}$ with Boltzmann constant $k_B = 1.38\cdot10^{-23}JK^{-1}$ and electron charge $e = 1.6\cdot10^{-19}C$.

=> Quasi-neutrality for any physical length $\lambda_D \ll L$ otherwise binary interaction should be considered.

Exercise: Compute the net charge of the shielding cloud.

The Plasma Parameter:

Average potential

$$\langle \Phi \rangle \sim \frac{e^2}{4\pi\epsilon_0 \langle r \rangle} \sim \frac{n_0^{1/3} e^2}{4\pi\epsilon_0}$$

Average kinetic energy:

$$\frac{1}{2} m_s \left\langle v^2 \right\rangle = \frac{3}{2} k_B T_s \equiv \frac{3}{2} m_s v_s^2$$

with v_s defined as the thermal speed of species s:

$$v_s = \left(\frac{k_B T_s}{m_s} \right)^{1/2}$$

with $m_e = 9.11 \cdot 10^{-31}$ kg and $m_p = 1.67 \cdot 10^{-27}$ kg. The plasma definition requires

$$\frac{n_0^{1/3} e^2}{4\pi\epsilon_0} \ll k_B T_s$$

such that

$$\frac{1}{4\pi} \ll n_0^{2/3} \frac{\epsilon_0 k_B T_s}{n_0 e^2} = n_0^{2/3} \lambda_{Ds}^2$$

with the result

$$\Lambda_s \equiv n_0 \lambda_{Ds}^3 \gg (4\pi)^{-3/2}$$

The parameter $\Lambda_D = n_0 \lambda_D^3$ is called the **plasma parameter**. Note that sometimes $1/\Lambda_s$ is called the plasma parameter.

The plasma definition implies

$$\Lambda = n_0 \lambda_D^3 \gg 1 \tag{1.6}$$

which is implies that the number of particles in a Debye sphere $N = \frac{4\pi}{3} n_0 \lambda_D^3$ is much larger than unity. This is consistent with the shielding. A considerable shielding of individual charges can occur only on the Debye length if there are sufficient charges in the Debye sphere of each individual particle.

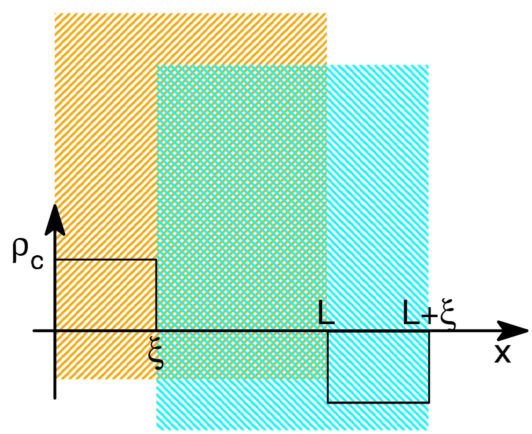
Exercise: Calculate the electron thermal speed, Debye length, and the plasma parameter for
(a) a tokamak plasma with $T_e = 10^8$ K, n_0 $=10^{19}$ m^{-3}
(b) the tail magnetosphere with $T_e = 10^7$ K, n_0 $=10^6$ m^{-3}
(b) the ionosphere with $T_e = 10^3$ K, n_0 $=10^{12}$ m^{-3}
(b) the solar atmosphere with $T_e = 10^4$ K, n_0 $=10^{20}$ m^{-3}
(b) a laser fusion plasma with $T_e = 10^7$ K, n_0 $=10^{29}$ m^{-3}

Exercise: Can a plasma be maintained at temperatures of $T_e = 100$ K. Hint: Calculate the density limit using the plasma parameter and explain

Exercise: $\Lambda \propto {n_0}^{-1/2} T^{3/2}$ While the dependence on temperature seems intuitively clear the density dependence appears odd because lower densities mean less particles and less shielding. Why does the plasma parameter improve (increase) with decreasing density?

1.2.3 Plasma Frequency

Consider an infinite slab of electrons and ion with a width of L (in x) and particle density of n_0. Assume that the electrons are displaced by a small distance $\xi \ll L$ in the x direction. This creates two regions of nonzero charge density:



The surplus charge generates an electric field E along the x direction. Using Poisson's equation in one dimension

$$\partial_x E = \frac{1}{\epsilon_0} \rho_c$$

and integrating over x yields

$$E = \frac{e n_0}{\epsilon_0} \xi$$

in the region between $x = \xi$ and $x = L$.

Evaluate force on the electron slab:

Charge per unit area: $-en_0L$ => Force: $F = -\frac{e^2n_0^2}{\epsilon_0}L\xi$

Here we have neglected the small contributions from the regions where the electrons and ions don't overlap. Mass of the electron per unit area: m_en_0L

such that Newton's law yields

$$-\frac{e^2n_0^2}{\epsilon_0}L\xi = m_en_0L\frac{d^2\xi}{dt^2}$$

which is the equation for a harmonic oscillator

$$\frac{d^2\xi}{dt^2} + \frac{e^2n_0}{\epsilon_0m_e}\xi = 0$$

with the frequency

$$\omega_{pe} = \left(\frac{n_0e^2}{m_e\epsilon_0}\right)^{1/2} \tag{1.7}$$

where ω_{pe} is called the electron plasma frequency.

In analogy we can define the ion plasma frequency as

$$\omega_{pi} = \left(\frac{n_0Z^2e^2}{m_i\epsilon_0}\right)^{1/2} \tag{1.8}$$

and the total plasma frequency as $\omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2$. Note that $\omega_{pe}^2 \gg \omega_{pi}^2$ such that $\omega_p^2 \approx \omega_{pe}^2$.

Exercise: Calculate the plasma frequencies for the examples of the exercise in the plasma parameter section.

The product of plasma frequency and Debye length is the thermal speed:

$$\omega_{ps}\lambda_{Ds} = \left(\frac{k_BT_s}{m_s}\right)^{1/2} = v_s$$

=> Plasma period is the time that a thermal particle needs to travel the distance of a Debye length!

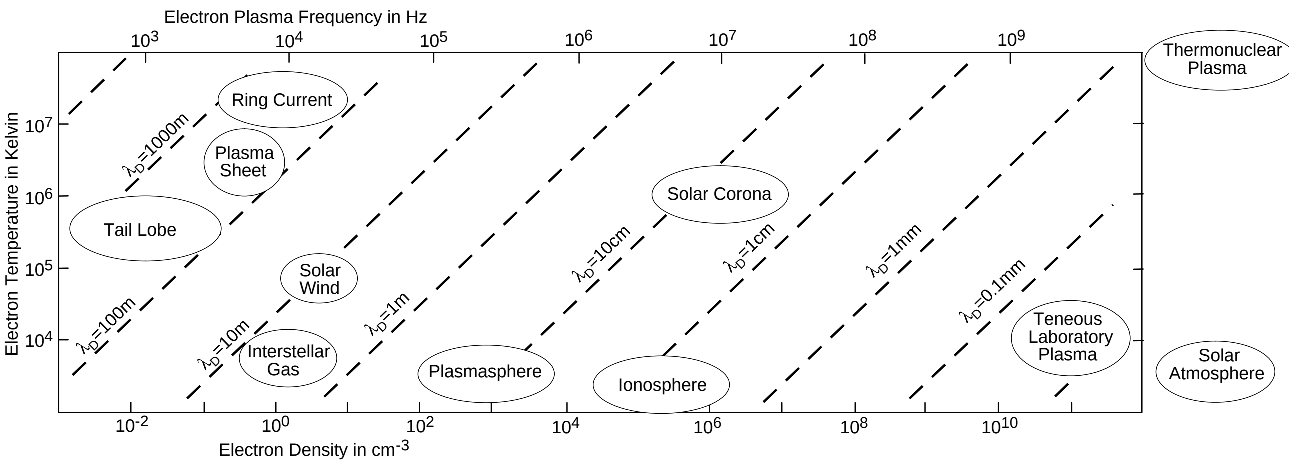
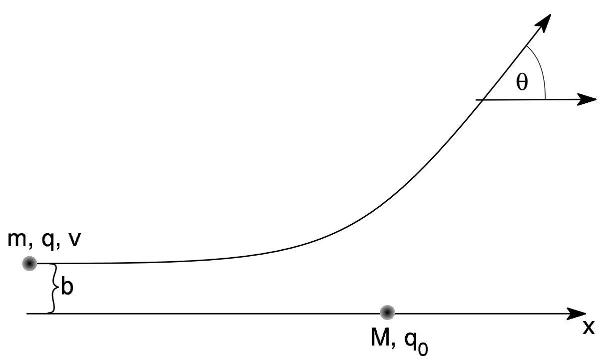


Figure 1.1: Plasma parameters.

1.3 Coulomb Cross section for Momentum Exchange

Scattering of momentum reduces the momentum of a stream of particles and is therefore generates resistance for a stream particles. To measure the resistance we need the total cross section for the momentum scattering.

Assume scattering of a particle with mass m, charge q, and velocity v by a particle with mass $M \gg m$ and charge q_0. The impact parameter is b.



Mechanics:

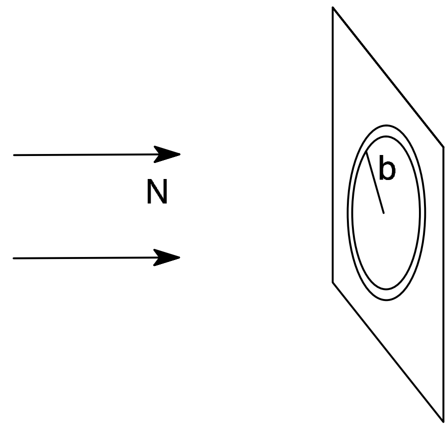
$$\cot \theta/2 = 4\pi\epsilon_0 \frac{bmv^2}{qq_0} \tag{1.9}$$

Exercise: Derive the above equation and calculate the the distance of the closest possible approach.

The momentum along x after the scattering is $p' = p\cos\theta$ such that the momentum change is

$$\Delta p = p\,(1 - \cos\theta)$$

Let N be the number of particles per time unit and unit area: $N = \frac{No\,Particles}{time\,unit\,area}$. Then $dN = N 2\pi b db$ particles are scattered into the the range $[\theta, \theta + d\theta]$.



Definition: Differential cross section for momentum exchange

$$d\sigma_p = \frac{dN}{N}\frac{\Delta p}{p} = 2\pi b db\,(1 - \cos\theta) \tag{1.10}$$

=> total cross section:

$$\sigma_{p,tot} = \int_{\theta_{min}}^{\pi} 2\pi b \frac{db}{d\theta}\,(1 - \cos\theta)\,d\theta \tag{1.11}$$

with

$$\begin{aligned} b &= A\cot\theta/2 \\ \frac{db}{d\theta} &= -\frac{A}{2\sin^2\theta/2} \end{aligned}$$

and $A = qq_0/(4\pi\epsilon_0 m v^2)$ we obtain

$$\begin{aligned} \sigma_{p,tot} &= -\pi A^2 \int_{\theta_{min}}^{\pi} (1 - \cos\theta)\,\frac{\cos\theta/2}{\sin^3\theta/2} d\theta \\ &= 4\pi A^2 \ln\frac{1}{\sin\theta_{min}/2} \end{aligned}$$

and for $\theta_{min} \ll 1$ =>

$$\sigma_{p,tot} = 4\pi A^2 \ln\frac{2}{\theta_{min}}$$

Finally we can express θ through the scattering parameters by noting that for small values of θ we have $\cot\theta_{min}/2 = 2/\theta_{min} = 4\pi\epsilon_0 b_{max} m v^2/(q q_0)$ such that3

$$\sigma_{p,tot} = 4\pi A^2 \ln\left[4\pi\epsilon_0 b_{max}\frac{mv^2}{qq_0}\right] \tag{1.12}$$

Here the expression $\ln\left[4\pi\epsilon_0 b_{max}\frac{mv^2}{qq_0}\right]$ is called the Coulomb logarithm.

Note that $\sigma_{p,tot} \sim electr\ resistance$.

For $b_{max} \to \infty$ (or $\theta_{min} \to 0$) => $\sigma_{p,tot} \to \infty$

Conclusion: The simple model of Coulomb scattering yields an inconsistency. The total cross section diverges through the contribution of large numbers of particle with large impact parameters.

However, at distance larger than the Debye length the coulomb potential is shielded.

=> $b_{max} = \lambda_D = \left(\frac{\epsilon_0 k_B T}{n_0 e^2}\right)^{1/2}$

Let us also consider a hydrogen plasma with $|qq_0| = e^2$ and $mv^2/2 = \frac{3}{2}k_B T$:

$$\begin{aligned}\ln\left[4\pi\epsilon_0 b_{max}\frac{mv^2}{qq_0}\right] &= \ln\left[\lambda_D\frac{4\pi\epsilon_0 3 k_B T n_0}{n_0 e^2}\right]\\ &= \ln\left[12\pi\lambda_D^3 n_0\right] = \ln\left[12\pi\Lambda\right]\end{aligned} \tag{1.13}$$

=> Coulomb logarithm is the logarithm of the plasma parameter!

No coincidence: The plasma parameter (and powers thereof) is the only possibility to create a dimensionless parameter that is a function of m, e, n, and T.

1.4 Collision frequency

The collision frequency is equal to the flux of particles vn_0 multiplied with the total cross section $\sigma_{p,tot}$:

$$\begin{aligned}\nu_c(v) &= v n_0 \sigma_{p,tot}\\ &= \frac{n_0 e^4}{4\pi\epsilon_0^2 m^2 v^3}\ln\left[12\pi\Lambda\right]\end{aligned}$$

Consider Maxwell distribution function:

$$f(\mathbf{v}) = n_0\left(\frac{m}{2\pi k_B T}\right)^{3/2}\exp\left(-\frac{mv^2}{2k_B T}\right)$$

Integral over velocity space yields average $\langle v^3\rangle = 8\sqrt{\frac{2}{\pi}}\left(\frac{k_B T}{m}\right)^{3/2}$such that

$$\nu_c = \sqrt{\frac{\pi}{2}}\frac{n_0 e^4}{32\pi\epsilon_0^2 m^{1/2}(k_B T)^{3/2}}\ln\left[12\pi\Lambda\right] \tag{1.14}$$

Exercise: Compute $\langle v^3 \rangle$

It is interesting to compare collision and plasma frequency:

$$\begin{aligned}\frac{\nu_c}{\omega_p} &= \left(\frac{n_0 e^2}{m\epsilon_0}\right)^{-1/2} \sqrt{\frac{\pi}{2}} \frac{n_0 e^4}{32\pi\epsilon_0^2 m^{1/2} \left(k_B T\right)^{3/2}} \ln\left[12\pi\Lambda\right] \\ &= \sqrt{\frac{\pi}{2}} \frac{n_0^{3/2} e^3}{32\pi n_0 \left(\epsilon_0 k_B T\right)^{3/2}} \ln\left[12\pi\Lambda\right] \\ &= \sqrt{\frac{\pi}{2}} \frac{1}{32\pi} \frac{1}{\Lambda} \ln\left[12\pi\Lambda\right]\end{aligned}$$

For $\Lambda \gg 1$ => $\nu_c/\omega_p \ll 1$

Binary collisions are less important than collective plasma effects!!

Exercise: Compute the mean free path and the ratio of the mean free path to the Debye length.

1.5 Plasma in a Fluid Limit

$$\begin{aligned}\lambda_D &= \left(\frac{\epsilon_0 k_B T}{n_0 e^2}\right)^{1/2} \\ \omega_p &= \left(\frac{n_0 e^2}{m\epsilon_0}\right)^{1/2}\end{aligned}$$

Individual particles: m, e

Consider limit of $m,\ e \ \rightarrow\ 0$

Conserved properties:

- mass density: mn_0
- charge density: en_0
- kinetic energy density: $n_0 k_B T$

=> $n_0 \sim 1/m$, $e \sim m$, and $T \sim 1/n_0 \sim m$

In this limit discreteness vanishes and fluid-like properties survive.

$$\begin{aligned}\lambda_D &= \left(\frac{\epsilon_0 k_B n_0 T}{n_0^2 e^2}\right)^{1/2} \\ \omega_p &= \left(\frac{n_0^2 e^2}{m n_0 \epsilon_0}\right)^{1/2}\end{aligned}$$

Debye length and plasma frequency remain unchanged.

Plasma parameter: $\Lambda = n_0 \lambda_D^3 \to \infty$

Collision frequency: $\nu_c \to 0$

Exercise: Determine the fluid limit for the plasma parameter, collision frequency, thermal speed, and gyro frequency eB/m. Discuss the results

1.6 Basic plasma equations

1.6.1 Maxwell's Equations

In general magnetic and electric fields are determined by Maxwell's equations, corresponding boundary conditions and the source (charges and currents) distributions.

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho_c \tag{1.15}$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j} \tag{1.16}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{1.17}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{1.18}$$

where $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic fields.

$c = 3 \cdot 10^8 ms^{-1}$, $\epsilon_0 = 8.85 \cdot 10^{-12} Fm^{-1}$, and $\mu_0 = 4\pi \cdot 10^{-7} Hm^{-1}$. Sometimes it is convenient to express the electromagnetic fields in terms of an electric potential Φ and a vector potential $\mathbf{A}$ such that

$$\begin{aligned}\mathbf{E}(\mathbf{r}, t) &= -\nabla \Phi(\mathbf{r}, t) - \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} \\ \mathbf{B}(\mathbf{r}, t) &= \nabla \times \mathbf{A}(\mathbf{r}, t)\end{aligned}$$

which requires to solve the electromagnetic field equations for the potentials for instance in the form

$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = \frac{1}{\epsilon_0}\rho_c(\mathbf{r},t) \quad (1.19)$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = \mu_0\mathbf{j}(\mathbf{r},t) \quad (1.20)$$

where Φ and $\mathbf{A}$ satisfy the Lorentz gauge $\partial\Phi/\partial t + c^2\nabla\cdot\mathbf{A} = 0$.

Exercise: Derive the equations for the scalar and the vector potentials using the Lorentz gauge.

Exercise: Derive the equations for the scalar and the vector potentials using the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$. Are the equations for the potential still valid for the Coulomb gauge?

1.6.2 Lorentz Equations of Motion

In electromagnetic fields the motion of charged particles is determined by the fields through the equations of motion

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i \quad (1.21)$$

$$\frac{d\mathbf{v}_i}{dt} = \frac{q_i}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B})_{\mathbf{r}_i,t}. \quad (1.22)$$

If the forces are external, the corresponding equations of motion are complete and the particle motion is often called test particle motion. However, if the other particles contribute to the force on a particular particle the forces have to be evaluated. The electromagnetic forces in a plasma depends on the current and charge densities which are determined by the **collective particle interaction.**

In a plasma the number of particles in a physical system is usually rather large. In addition the overwhelming majority of problems deal with the collective particle behavior rather than the individual one. Discrete particle dynamics is important in some areas of plasma physics for sufficiently small ('microscopic') length or time scales.

Plasma physics combines elements from various other area of theoretical physics (mechanics, statistical mechanics, E+M, and QM).

1.6.3 Examples of Kinetic Equations

To describe a plasma one can solve the coupled system of Maxwell's equations and the particle equations of motion. However, there are more efficient methods to solve the plasma dynamics using the above approximations.

As an example consider the single particle distribution function $f(\mathbf{r},\mathbf{u},t)$ which gives the density of particles in the six-dimensional space consisting of the set of ordinary coordinates and velocities $(\mathbf{r},\mathbf{u})$.

The number of particles in the interval given by $[\mathbf{r}, \mathbf{r}+d\mathbf{r}]$ and $[\mathbf{u}, \mathbf{u}+d\mathbf{u}]$ is given by $f(\mathbf{r}, \mathbf{u}, t)d\mathbf{r}d\mathbf{u}$. The single particle distribution function satisfies the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}} f = \left.\frac{\partial f}{\partial t}\right|_{collisions} \tag{1.23}$$

Here the rhs of the equation considers collisional effects.

TO solve the plasma equations one needs to evaluate charge and current density from1.23

$$\begin{aligned} q_s n_s &= q_s \int_{-\infty}^{\infty} d^3v f_s(\mathbf{x}, \mathbf{v}, t) \\ q_s n_s \mathbf{u}_s &= q_s \int_{-\infty}^{\infty} d^3v \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t) \end{aligned}$$

and solve Maxwell's equations.

In the case of thermal equilibrium f assumes locally a Maxwell distribution in velocity space and the collision term on the rhs of 1.23 vanishes. Equation 1.23is a fluid (advection) equation though in a 6 dimensional space. The lhs can be interpreted as the total derivative of $f(\mathbf{r}, \mathbf{u}, t)$ along a trajectory given by the 6 dimensional velocity $\mathbf{v}^{(6)} = (\mathbf{v}, \frac{\mathbf{F}}{m})$ where $\mathbf{F}$ is the Lorentz force.

Often collisions can be neglected almost everywhere except for small regions in space. In the absence of collisions the total derivative along the path determined by $\mathbf{v}^{(6)}$ is

$$\frac{df}{dt} = 0$$

in analogy of the advection equation in ordinary space

$$\partial f/\partial t + \mathbf{v}\cdot\nabla f = df/dt$$

Illustration: Consider an observer is moving with velocity $\mathbf{v}$ and a distribution function $f(\mathbf{x}, t)$. The observed distribution function f at time t_0 is

$f_0 = f(\mathbf{x}_0, t_0)$

and at $t_1 = t_0 + dt$

$f_1 = f(\mathbf{x}_1, t_1) = f(\mathbf{x}_0 + \mathbf{v}dt,\, t_0 + dt) = f(\mathbf{x}_0, t_0) + dt\frac{\partial f}{\partial t} + \mathbf{v}dt\cdot\nabla_{\mathbf{x}} f$ such that the total change in the distribution function is

$$df = f_1 - f_0 = dt\left(\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}} f\right)$$

and the total time derivative

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}} f$$

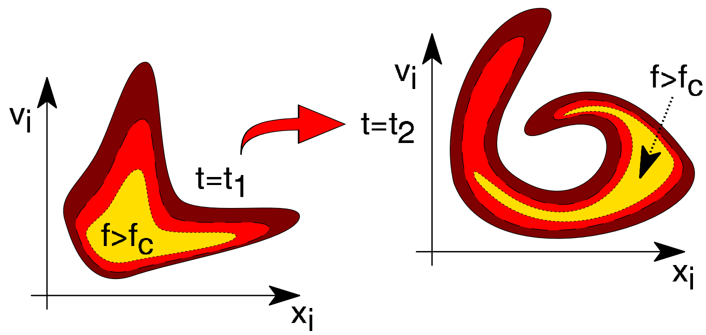


Figure 1.2: Illustration of the evolution of a collisionless distribution conserving the area inside contours of constant f.

$df/dt = 0$ implies that the change along the trajectory given by $\mathbf{v}$ is 0. In the present case of a plasma the $\mathbf{v}^{(6)} = (\mathbf{v}, \frac{\mathbf{F}}{m})$ is the velocity of the plasma in the 6 dimensional space given by $\mathbf{x}^{(6)} = (\mathbf{x}, \mathbf{v})$.

This implies that the value of f is conserved along this trajectory in the six-dimensional phase space. In particularly any maximum of the distribution remains exactly the same. Similarly in the collsionless case one would expect that no plasma can be created or annihilated (except if the is an appropriate collision which implies that f should also satisfy a continuity equation of the form

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f) + \nabla_{\mathbf{v}} \cdot \left(\frac{\mathbf{F}}{m} f \right) = 0$$

This particularly implies that the six-dimensional divergence of $\mathbf{v}^{(6)}$ is 0 or in other words that the evolution in six-dimensional space is incompressible, that means that the phase space volume of a contour with any constant value of f is conserved during the evolution as illustrated in Figure 1.2.

Exercise: Consider an ordinary continuity equation $\partial n/\partial t + \nabla \cdot (\mathbf{v}n) = 0$. The number of particles in an arbitrary volume is $N = \int_V n d^3 r$. Show that the number of particles changes only due to particle flux trough the surface of the volume V.

The set of the collisionless Boltzmann equation 1.23 complemented by Maxwell's equations is called Vlasov equations. More properties of kinetic equations (equilibria, collision operators etc.) and the origin of the Boltzmann equation will be discussed in the later chapter on kinetic plasma equations.

The collision term on the rhs can consider many different physical or chemical processes. Chemical reactions, ionization or recombination, friction, diffusion, and energy exchange collisions are contained in the collision term. Details depend on the corresponding processes.