1. Amdahl's law.

Consider parallel processing with shared memory. The number of parallel processor is 254. Explain why Amdahl's law for this situation is the same as for vector processing. Using Amdahl's law, plot the speed-up in execution time (on a log base 10 scale) as a function of the fraction p of the program that is parallelized. How much of the program (in terms of operations) must be parallelized to achieve speedups of 3, 10, and 250?

2. Parallel Processing.

The total execution time of a fully parallelized simulation code ($p = 1$) of size N (no. of elements or grid points) on N_p parallel processors in a distributed memory environment is

$$\tau_{tot} = 2d \frac{N^{2/3}}{N_p^{\alpha}} \tau_c + \frac{N}{FN_p} \tau_s$$

where $N^{2/3}/N_p^{\alpha}$ is the number of elements which require inter-processor communication, τ_c is the communication time required for one element, F is the execution efficiency for an individual processor, and τ_s is the time for the computation of an individual element. Assume $\tau_c = 2\tau_s = 10^{-6}$, $F = 0.95$, and $N = 10^9$.

a) Different domain decompositions yield $\alpha = 2/3$, $1/2$, and 0 (these values correspond to a 3D, 2D, and 1D decomposition of a three-dimensional grid). Plot the ratio of the total communication time and the total computation time as a function of N_p for 2 to 10000 processors. Use a (base10) logarithmic with an appropriate scale for the vertical axis. Discuss shortly your results for the 3 domain decompositions.

b) A parallelization becomes inefficient when a considerable fraction of the total time is spend for inter-processor communication. Determine the number of processors for each of the 3 values of α for which half of the total time is spent on communication. Compare the total execution times for the three cases.

3. Amdahl's law for nested parallel processing.

Consider a computer environment with N_n parallel nodes where each node has N_m parallel computing cores such that the total number of computing processors is $N_p = N_n N_m$. Each node has shared memory for the processors on the node. However, between nodes memory is distributed. Derive Amdahl's law for the speedup in this processing environment consisting of a hierarchy of distributed and shared memory. What is the maximum speedup? Assume a fixed number of total processors N_p. Is there an optimum distribution (ratio) for the N_n and N_m processors (for $N_m > 1$) in terms of the most efficient execution and what is this ratio? For each level assume that the fractions of operations that can be parallelized are P_n and P_m and fixed.

Please turn in the solutions to the homework on Monday 2/4/13.