4. Pressure/energy equation:

Starting from the energy equation (2.20) derive the pressure equation (2.21). For simplicity assume a scalar pressure, i.e., $\underline{\underline{\Pi}} = \underline{\underline{1}} p$, and zero source terms $Q^\rho = 0$, $\mathbf{Q}^p = 0$, and $Q^E = 0$. (Hint: Use the momentum and continuity equations to eliminate time derivatives of density and velocity in the energy equations).

5. Heat conduction: Starting from the pressure equation

$$\frac{1}{\gamma - 1}\left(\frac{\partial}{\partial t} p + \nabla \cdot p\mathbf{u}\right) = -(\underline{\underline{\Pi}} \cdot \nabla) \cdot \mathbf{u} - \nabla \cdot \mathbf{L} \tag{1}$$

derive an equation for the temperature T using an isotropic pressure (the viscosity $\underline{\underline{\mathbf{w}}} = 0$), $p = nkT$, and the heat flux is $\mathbf{L} = -\kappa \nabla T$. Note, to eliminate the time derivative of n use the continuity equation. Show that for vanishing velocity $\mathbf{u} = 0$ the equation becomes a diffusion equation for temperature.

6. Simple hyperbolic equation:

(a) Show that the second order PDE $\partial^2 u / \partial x \partial t = 0$ is hyperbolic.

(b) Demonstrate that the system

$$\frac{\partial u}{\partial t} - v = 0 \qquad \frac{\partial v}{\partial x} = 0$$

is equivalent.

(c) Use the method for multiple 1st order PDE's to demonstrate that it is hyperbolic and show that the characteristics are the x and t axes.

7. Time dependent flow:

The equations for one-dimensional isentropic inviscid flow are

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{u \partial \rho}{\partial x} + \frac{\rho \partial u}{\partial x} &= 0 \\ \frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} &= 0 \\ p &= k\rho^\gamma \end{aligned}$$

reduce this system of three dependent variables to two by eliminating p. Show that the system is hyperbolic and that the characteristics are given by $dx/dt = u \pm a$ with the sound speed $a^2 = \gamma p / \rho$.

Please turn in the solutions to the homework on Monday, 2/11/2013