**8.** Pressure/energy equation:
Consider the pressure equation for isotropic pressure in the absence of heat conduction

$$\frac{1}{\gamma-1}\left(\frac{\partial p}{\partial t}+\nabla\cdot p\mathbf{u}\right)=-p\nabla\cdot\mathbf{u}+Q^E$$

a) Assuming $h=p/\rho^\gamma$, demonstrate that the pressure equation combined with the continuity equation can be written as

$$\frac{\partial h}{\partial t}+\mathbf{u}\cdot\nabla h=R$$

and compute $R$ as a function of $Q^E$.

**9. a)** Demonstrate that the two-dimensional compressible irrotational steady state flow

$$\begin{aligned}\left(\frac{u^2}{a^2}-1\right)\frac{\partial u}{\partial x}+\left(\frac{uv}{a^2}\right)\frac{\partial u}{\partial y}+\left(\frac{uv}{a^2}\right)\frac{\partial v}{\partial x}+\left(\frac{v^2}{a^2}-1\right)\frac{\partial v}{\partial y} &= 0\\ -\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} &= 0\end{aligned}$$

are equivalent to the equations

$$-\frac{\gamma p}{\rho}\nabla\cdot\mathbf{u}+\mathbf{u}\cdot((\mathbf{u}\cdot\nabla)\mathbf{u}) = 0 \qquad (1)$$

$$\nabla\times\mathbf{u} = 0 \qquad (2)$$

for the velocity $\mathbf{u}=(u,v)$ with $a^2=\gamma p/\rho$.

**b)** Derive equation (1) using the steady state continuity (2.18), momentum (2.19), and pressure equation (2.20) for isotropic pressure $\underline{\underline{\Pi}}=p\underline{\underline{1}}$ (no source terms, no external force terms, and no heat conduction. Hint: Use the continuity and momentum equations to replace the $\nabla\rho$ and $\nabla p$ terms in the pressure equation).

**10.** Read section 3.5 of the manuscript, derive the Fourier transform of the Navier Stokes equations, and show that the determinant for the Fourier transform matrix is

$$(\sigma_x^2+\sigma_y^2)\left[i(u\sigma_x+v\sigma_y)+\frac{1}{R_e}(\sigma_x^2+\sigma_y^2)\right]=0$$

**11.** General Technique - 2nd Derivative Approximation
**a)** Use the general technique to determine the coefficients $a$ to $c$ and the leading error term in the following expression

$$\frac{d^2f}{dx^2}=af_{i-2}+bf_{i-1}+cf_i$$

**b)** Do the same for the expression

$$\frac{d^2f}{dx^2}=af_{i-1}+bf_i+cf_{i+1}+df_{i+2}+ef_{i+3}$$

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Please turn in the solutions to the homework on Monday, 2/18/2013