12 The equation $\partial\phi/\partial t - \alpha\partial^2\phi/\partial x^2 = 0$ is to be solved in the domain $0 \le x \le 1$, $t > 0$ with boundary conditions $\phi(0,t) = 0$, $\phi(1,t) = \phi_R$ and the initial condition $\phi(x,0) = 0$. Show via the separation of variables technique, that the solution is

$$\phi = \phi_R x + \sum_{k=1}^{\infty} \frac{2\phi_R (-1)^k \exp\left(-k^2\pi^2\alpha t\right)\sin(k\pi x)}{k\pi}.$$

13. Accuracy:

A three-level explicit discretization of $\frac{\partial T}{\partial t} - \alpha\frac{\partial^2 T}{\partial x^2} = 0$ can be written as

$$\frac{0.5T_j^{n-1} - 2T_j^n + 1.5T_j^{n+1}}{\Delta t} - \alpha\left[\frac{(1+d)(T_{j-1} - 2T_j + T_{j+1})^n}{\Delta x^2} - \frac{d(T_{j-1} - 2T_j + T_{j+1})^{n-1}}{\Delta x^2}\right] = 0$$

(a) Expand each term as a Taylor series to determine the truncation error of the complete equation for arbitrary values of d.

(b) Use the general technique to choose d as a function of $s = \alpha\Delta t/\Delta x^2$ so that the scheme is fourth-order accurate in Δx.

Hint: Terms involving $\partial T/\partial t$ (or higher derivatives of t) can be expressed as x derivatives by making use of $\partial T/\partial t = \alpha\partial^2 T/\partial x^2$. Terms with Δt in the expansion can be written as $\Delta t = s\Delta x^2/\alpha$ since the time integration parameter is usually s.

14. Simulation of the diffusion equation - SIM1:

Try to understand the simulation code sim1. Run the code for different parameters by varying the time step through s, and Δx (by means of nx). Plot results for $\Delta x = 0.1$ and 0.05 (fixed α ans $s = 0.5$) with the provided IDL program (or choice of a plotting routine but in a similar format as shown in class). To compared results change the output parameter for the different resolutions such that outputs are generated at the same physical time. Now change α to half the initial value. Generate again output at the same physical times (for $\Delta x = 0.05$). Understand and comment at which time levels output is generated depending on the altered parameters. If you change s to half its value (keeping α and Δx fixed) how do you need to change the output parameter to generate output at the same physical times? Finally run the program for $s = 1/6$ (again with output at the same physical times) How does the error change for different resolution and different values of s? What is the reason for the change in the error?

Please turn in the solutions to the homework on Monday, 2/25/2013