18. Stability analysis:

For the diffusion equation it seems rather appropriate to use a scheme

$$\frac{T_j^{n+1} - T_j^{n-1}}{2\Delta t} - \frac{\alpha(T_{j-1} - 2T_j + T_{j+1})^n}{\Delta x^2} = 0$$

because the centered time should give higher accuracy for the time derivative. Test the stability of this method. Is it worth an attempt to implement it?

19. Stability test

Use the program from problem 16 to test the stability of the scheme in problem 13 as a function of s for different values of the parameters d. Does the result for $d = 0$ agree with your analytic stability limit in Problem 15? How does the stability limit vary for different values of d including the particular choice of $d = 1 - \frac{1}{12s}$.

20. Weighted residual method:

The equation $d^2y/dx^2 + y = \left(1 - (5\pi/6)^2\right) \sin(5\pi x/6)$, subject to the boundary conditions $y(0) = 0$ and $y(1) = 0.5$, is to be solved in the domain $x \in [0,1]$ using the following methods of the weighted residuals

(a) Collocation,

(b) Galerkin.

Assume the approximate solution to be $y = a_1(x - x^3) + a_2(x^2 - x^3) + 0.5x^3$ with the base functions $\Phi_1 = (x - x^3)$ and $\Phi_2 = (x^2 - x^3)$.

Compare the computational solution with the exact solution $y = \sin(5\pi x/6)$.

21. Fivol and Laplaces equation:

Obtain solutions to Laplace's equation in the region introduced with the program fivol.f with this program for the following parameters:

(a) $r_W = 0.1$, $r_X = 4.00$, $r_Y = 1.00$, $r_Z = 0.10$, $\theta_{WX=0}$, and $\theta_{ZY} = 90$;
(b) $r_W = 0.1$, $r_X = 8.00$, $r_Y = 1.00$, $r_Z = 0.10$, $\theta_{WX=0}$, and $\theta_{ZY} = 90$.

For each case obtain results for the three grids, JMAX = KMAX = 6, 11, 21. Plot your results. Compare the accuracy and the rate of convergence with the results from the following table for the parameters: $r_W = 0.1$, $r_X = 1.00$, $r_Y = 1.00$, $r_Z = 0.10$, $\theta_{WX=0}$, and $\theta_{ZY} = 90$, $\lambda = 1.5$.

Grid	$\|\|\phi - \overline{\phi}\|\|_{rms}$	No of iterations for convergence
6×6	0.1326	15
11×11	0.0471	19
21×21	0.0138	51

Please turn in the solutions to the homework on Monday, 3/18/2013