

22. Quadratic FEM

For the finite element method, second derivatives in a partial differential equation are determined by the terms $l_{ij} = \int \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} dx$. Quadratic finite element base functions are given by $\phi_{A,B} = 0.5\xi(\xi \pm 1)$ on even nodes and for odd nodes they are $\phi = (1 - \xi^2)$. The transformation between x and ξ is given by

$$\xi_A = 2 \frac{x - 0.5(x_{j-2} + x_j)}{x_j - x_{j-2}}$$

$$\xi_B = 2 \frac{x - 0.5(x_{j+2} + x_j)}{x_{j+2} - x_j}$$

Compute the second derivative coefficients l_{ij} for even and for odd nodes. (Note: You don't have to compute all nonzero coefficients if you make use of symmetry properties.)

23. Modification of Fivol

Extend the program FIVOL to solve the equation

$$\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} - \alpha \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) = S$$

The linear terms can be included in a similar way as discussed for the finite volume method and first order derivatives. The source term S can be treated in the same way as the term $\partial q / \partial t$ which has been discussed for the first order derivatives. Test the program for the case $S = (\cos(2\theta) - \sin(2\theta)) / r^2$ by obtaining solutions for the same boundary conditions and parameter values as in homework 21 except for r_X which should be chosen as $r_X = 1.0$. Compare the solutions with the exact solution $\phi = (\sin \theta) / r$.

24. Program DUCT

To become familiar with the program run the program DUCT for grid resolutions of 6×6 , 11×11 , and 21×21 . determine the rms error for the finite differences and finite element methods (you can switch between them with a parameter). Tabulate and comment your results. Plot an example for the coarse and the fine grid spacing.

25. Select a class project. Try to formulate goals for the project and become familiar with the methodology. Provide a brief report (not more than three pages) on this progress with your project. Particularly list also any problems you may have encountered. This report as any future reports should demonstrate that you have indeed thought about the problem and made first progress (or not if documented by a corresponding discussion of the problems).