

36. The DuFort-Frankel method is given by

$$\frac{f_j^{n+1} - f_j^{n-1}}{2\Delta t} = \alpha L_{xx} f_j^n = \frac{\alpha}{\Delta x^2} (f_{j-1}^n - f_j^{n-1} - f_j^{n+1} + f_{j+1}^n)$$

a) Derive the truncation error.

b) Show that the amplification factor from the von Neumann stability analysis is given by

$$g = \frac{2s \cos k\Delta}{1 + 2s} \pm \frac{1}{1 + 2s} \sqrt{1 - 4s^2 + 4s^2 \cos^2 k\Delta}$$

c) Discuss the stability resulting from this amplification factor.

37. Implementation of (a) the DuFort-Frankel and (b) the Hopscotch (1D) schemes in program sim1:

Hopscotch: 1st stage

$$\frac{\Delta f_j^{n+1}}{\Delta t} = \alpha L_{xx} f_j^n \quad j+n = \text{even}$$

2nd stage:

$$\frac{\Delta f_j^{n+1}}{\Delta t} = \alpha L_{xx} f_j^{n+1}, \quad j+n = \text{odd}$$

(a) First derive the algebraic equations needed for the DuFort-Frankel and the Hopscotch scheme and explain what changes you need to apply to sim1.f for the implementation. Carry out the implementation.

(b) Test the schemes first for a set of standard parameters used for the FTCS scheme with a grid number of 21 points and compare the result to the FTCS. Plot the results.

(c) Then use increasingly larger time steps for the new schemes, report your observations, and plot select results. Pay attention to the fact that the time steps should be chosen such that the final output is consistent with the analytic output for the final time.

38. By now you should have some basic/preliminary implementation of the code you are using for the project. Report on this progress and be detailed, i.e., provide select results or detailed descriptions/documentation on any problems in your progress.

Please turn in the solutions to the homework on Monday, 4/22/2013