The following lists a number of projects for the simulation course. The objective of these projects is to provide an opportunity to become more familiar with a particular aspect of numerical modeling. The projects only provide a frame to develop and understand certain aspects of a numerical method. The emphasis can be on the physics or on special numerical aspects. As mentioned the outline only provides a frame in which to work; the specific goals, applications etc can be altered if suitable or desired. Any other numerical projects may be suggested and will be accommodated if they are suitable for this class and fit into the time frame for the completion of the projects.

The projects do not imply a fully closed task with a particular fixed solution. Rather, it is encouraged to be creative, follow you own ideas and develop from those your understanding of a numerical/simulation problem. The project presentation does not expect a certain result. It should demonstrate the degree to which you progressed into the project, show the results which you have obtained, and should be concise. There will be 30 minutes for a presentation of the project and it is expected that you provide a report summarising you progress. This should be accompanied by any programs which you have developed for the project.

1. One-dimensional MHD simulation:

Magnetohydrodynamic simulations are a common tool for many laboratory and space plasma systems. They can be used to study basic plasma waves and nonlinear plasma processes. They are particularly powerful in demonstrating important plasma physics. In many cases basic physics only requires one dimension (linear and nonlinear waves, shocks, equilibria). A one-dimensional simulation coded is provided. The code uses a leapfrog and a Lax-Wendroff scheme and has the option to use a variety of initial conditions. Possible directions :

- A study and analysis of various MHD waves and discontinuities
- Implementation of different initial conditions (note that boundary conditions need to be consistent)
- Implementation of additional physics (Hall effect, gravity, etc.)

2. Two-dimensional fluid instabilities:

Suggested is to study the Kelvin-Helmholtz (KH) instability with two-dimensional fluid simulations. Macro-instabilities are of large importance in many fluids and gases. Typical are instabilities which are driven by a fluid flow (Kelvin-Helmholtz) or in the presence of gravity by the presence of heavier material on top of a lower density fluid (Rayleigh Taylor). A two-dimensional MHD code is available to study such instabilities. Note that the Kelvin-Helmholtz mode does not require the magnetic field. Possible directions:

- Rewrite the code to eliminate the magnetic terms and study the KH instability with the modified code as a function of parameters such as viscosity, initial velocity, shear layer thickness.
- Rewrite the code and implement a gravitational term to study Rayleigh Taylor instability or gravity waves (Switch off the magnetic field). Determine the influence of the gravitational term on the processes that you study.

3. Flux corrected transport (fct):

A typical property of many fluids is the steepening of waves to form shock waves. Such shock waves are also present in front of objects moving with supersonic velocities. In an ideal fluid description shock waves are discontinuities across which density, temperature, and velocity change abruptly. In a numerical simulation discontinuities are not possible and large changes on the grid scale are not desirable because they introduce large numerical errors (dispersion and diffusion). In practise a diffusive scheme will spread the discontinuous solution over many grid spacings thus reducing errors corresponding to numerical dispersion (on the grid scale). But large diffusion introduces also nonphysical errors. A scheme with very little diffusion will develop large oscillations on the grid scale because of numerical dispersion. A better solution can be obtained if diffusion is localized on in the region where the shock forms. This is the purpose of so-called flux corrected transport schemes. A good description and example of an fct algorithm can be found in Fletcher, Volume 2.

- Implement the fct algorithm in a simple fluid 1D simulation. A code for the 1D fluid simulation can be provided. Test the algorithm by changing the various parameters. compare the results to other schemes without the fct.

4. Three-dimensional heat conduction:

Diffusion and heat conduction problems occur in many places in nature. Diffusion of a trace gas or pollutant in air or water and diffusion or conduction of heat in various applications which have to do with insulation, temperature balance etc. Here in Fairbanks an important application is heat conduction in permafrost. Buildings on permafrost can have quite unfortunate results. The distribution of the ground temperature and its evolution in the vicinity of a building is a typical heat conduction problem.

- Develop a three-dimensional heat conduction or diffusion code and apply it to a typical situation as outlined above. For the situation you need to identify the material constants (conduction or diffusion coefficient) and expand a diffusion or heat conduction code to three dimensions.

5. Nonlinear Poisson's equation:

There are many fundamental systems or situations which are governed by elliptic equations and particularly by the Poisson equation. The time independent diffusion equation is one example. Poisson's equation is central for electrostatic problems. Two-dimensional electromagnetic equilibria are determined by Poisson's equation where the potential is a component of the vector potential and the source is the current density which in turn can be written as a function of the potential. Thus a scheme to solve the nonlinear Poisson equation has a wide range of applications. Basis for the scheme are the programs fivol or duct.

- Modify the base program to include a nonlinear source term to reflect the charge distribution or the current distribution as a function of the corresponding potential. Change the boundary condition to the physical situation and obtain solution for varying boundary conditions.

- Applications can be the solution to the electrostatic potential with the potential prescribed at the boundary or the solution to the component of the vector potential with the current prescribed as a function of the vector potential. In both cases the source is an exponential function of the potential A, i.e., s=exp(dA). The basic equation will be provided as well as guidance to choose the boundary conditions.

6. Von Weizsäcker's Cow

There has been a suggestion (presumable by von Weizsäcker) that spiral galaxies form as a result of differential rotation, i.e, it does not matter much what the initial distribution of matter in an early galaxy is and the differential rotation will generate the spiral shape. So one could conceivably consider the initial shape as one of a cow and then the rotation form this shape into a spiral galaxy. Here the objective is to write a two-dimensional code which solves the continuity equation in two dimensions. Program an initial state that is remotely reminiscent of a cow (or a moose if you prefer an Alaskan motive). Use this initial configuration and a prescribed initial velocity profile which describes a differential rotation and is divergence free $\nabla \cdot \mathbf{v} = 0$ such that the resulting dynamics is incompressible. Test properties of the code (mass conservation etc.).

7. Diffusion in with a varying heat or electrical conductivity

Find solutions to the stationary heat conduction equation of the form $\nabla \cdot (\lambda(x, y)\nabla T) = 0$. This equation can be used for various applications. For instance in a case where the heat conduction coefficient has variation on can imagine that thermal leakage through a surface can be significantly enhanced if the heat conduction coefficient is non-uniform. A second interpretation of this equation is in terms of a nonuniform electrical conductance. In this case T must be interpreted as the electrostatic potential ϕ. Keeping the potential constant at two opposite boundaries (similar to the heat conduction problem) the equation above is the stationary condition for the electric current $\nabla \cdot \mathbf{j} = 0$ with $j = \sigma \mathbf{E}$. If there are isolated depletions or enhancements of the electrical conductivity, how much can the local electric field be amplified and what is the electric charge distribution ($\nabla \cdot \mathbf{E}$). How is the current changed in the presence of conductivity variations?