21. **Stability of the Z pinch:** The pressure and magnetic field profile of the Z pinch is

\[
B_\theta = \frac{\mu_0 j_0}{2} r \quad \text{and} \quad p = -\frac{\mu_0 j_0^2}{4} r^2 + p_0 \quad \text{for} \quad r \leq r_c = \left(\frac{4p_0}{\mu_0 j_0^2}\right)^{1/2}
\]

Assume perturbations with \(\partial/\partial \theta = 0\) and apply the stability criterion derived in class to the equilibrium to determine its equilibrium properties. How do these change if a constant pressure \(p_c\) is superimposed.

22. **Stability of the Harris Sheet:** To determine the stability criterion for the Harris sheet equilibrium \(B = B_0 \tanh \frac{x}{L} e_y\) use the Potential

\[
U = \frac{1}{2} \int_V \left[ \gamma p_0 \left( \nabla \cdot \xi \right)^2 + \frac{1}{\mu_0} \left( \nabla \times (\xi \times B_0) \right)^2 \right.
\]

\[
+ \xi \cdot \nabla p_0 \nabla \cdot \xi - \frac{1}{\mu_0} \left( \xi \times (\nabla \times B_0) \right) \cdot \nabla \times (\xi \times B_0) \left. \right] dx
\]

for \(\partial \xi / \partial y = 0\). (Perturbations at the boundary at \(x = \infty\) are 0.)

(a) Show that

\[
\nabla \times (\xi \times B_0) = B \left[ \frac{\partial \xi_x}{\partial y} e_x - \left( \frac{\partial \xi_z}{\partial z} + \frac{1}{B} \frac{\partial \xi_x}{\partial x} \right) e_y + \frac{\partial \xi_z}{\partial y} e_z \right]
\]

\[
\xi \times (\nabla \times B_0) = \frac{\partial B}{\partial x} (\xi_y e_x - \xi_x e_y)
\]

(b) Bring \(U\) into the form

\[
U = \frac{1}{2} \int_V \left[ a_{11} \left( \nabla \cdot \xi \right)^2 + 2a_{12} \xi_x \nabla \cdot \xi + a_{22} \xi_x^2 \right] dx
\]

and show that

\[
a_{11} = \gamma p + \frac{B^2}{\mu_0}
\]

\[
a_{12} = \frac{dp}{dx} + \frac{B}{\mu_0} \frac{dB}{dx}
\]

\[
a_{22} = \frac{1}{\mu_0} \left( \frac{dB}{dx} \right)^2 + \frac{dp}{dx} \frac{1}{B} \frac{dB}{dx}
\]

(c) Evaluate the coefficients for the Harris sheet and determine and discuss the stability properties for the assumed perturbation.

23. **Project:**

Provide a report on your progress with the class project. Summarize the remaining tasks that you expect before you can actually present the project.

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Please turn in the solutions to the homework on Thursday day, 4/26/2012