30. Obtain solutions using the program duct \((b/a = 1)\) for a \(21^2\), \(41^2\), \(81^2\), \(161^2\), and \(321^2\) grid both for the finite element and the finite difference method.

a) For each grid determine through iteration the optimum value of the iteration (SOR) parameter \(\lambda_{opt}\), and the corresponding number of iterations \(N_{iter}\). Help: You need to determine to evaluate \(\lambda_{opt}\) to an accuracy of \(\sim 10^{-3}\) (or better for higher grid numbers) and examine if you can use the value of \(2 - \lambda_{opt}\) as a predictor for the next higher grid number.

b) Plot \(\ln N_{iter}\) versus \(\ln N\) (with \(N = N_x \cdot N_y\)) for the fde and fem methods. What asymptotic \((N \to \infty)\) scaling do you obtain for \(N_{iter}\) for the respective discretization?

c) Plot \(\ln (2 - \lambda_{opt})\) versus \(\ln N\) for the fde and fem methods. What asymptotic \((N \to \infty)\) scaling do you obtain for the value of \(2 - \lambda_{opt}\).

31. Use the same grid sequence (as in problem 30) for the program duct with finite differences and determine the number of iterations for Jacobi and Gauss-Seidel (GS) iterations. Plot the number of iterations \(\ln N_{iter}\) versus the number of grid points \(\ln N\). What scaling do you obtain now for \(N_{iter}\) and \(N \to \infty\) and how does the numerical effort for the two methods compare to the SOR method in problem 30 (How do \(N_{iter,GS}/N_{iter,SOR}\) and \(N_{iter,J}/N_{iter,SOR}\) scale with the total number of grid points)?

32. Report on the progress of your work on the project.

Please turn in the solutions to the homework on Monday, 4/8/2013